BK BIRLA CENTRE FOR EDUCATION

SARALA BIRLA GROUP OF SCHOOLS
SENIOR Secondary Co-Ed DAY CUM BOYS' RESIDENTIAL SCHOOL

PRE-BOARD EXAMINATION-1 (2023-24)

MATHEMATICS (041)

Class: XII Science | Duration: 3 Hrs
Date: 13-12-2023 | Max. Marks: 80
Admission number: ___________ | Roll number: __

General Instructions:

1. This question paper has 5 sections A, B, C, D and E.
2. Section A has 20 MCQs carrying 1 mark each.
3. Section B has 5 questions carrying 2 marks each.
4. Section C has 6 questions carrying 3 marks each.
5. Section D has 4 questions carrying 5 marks each
6. Section E has 3 case based integrated units of assessment (04 marks each) with subparts of the values 1, 1 and 2 marks each respectively.
7. All questions are compulsory. However, an internal choice in 2 Qs of 5 marks, 2Qs of 3 marks and 2 Qs of 2 marks has been provided. An internal choice has been provided in the 2 marks questions of Section E.
8. Draw neat figures wherever required. Take $\pi = \frac{22}{7}$ wherever required if not stated.

SECTION – A

1 If for a square matrix A, $A^2 - 3A + I = 0$ and $A^{-1} = xA + y\,I$ then the value of $x + y$ is

(A) -2 (B) 2 (C) 3 (D) -3

2 If $|A| = 2$ where A is 2×2 matrix, then $|4\,A^{-1}|$ equals

(A) 4 (B) 2 (C) 8 (D) $\frac{1}{32}$

3 Let A be a 3×3 matrix such that $|adj\ A| = 64$. Then $|A|$ is equal to

(A) $8\ only$ (B) $-8\ only$ (C) 64 (D) $8\ or -8$

4 If $A = \begin{bmatrix} 3 & 4 \\ 5 & 2 \end{bmatrix}$ and $2A + B$ is a null matrix, then B is equal to

(A) $\begin{bmatrix} 6 & 8 \\ 10 & 4 \end{bmatrix}$ (B) $\begin{bmatrix} -6 & -8 \\ -10 & -4 \end{bmatrix}$ (C) $\begin{bmatrix} 5 & 8 \\ 10 & 3 \end{bmatrix}$ (D) $\begin{bmatrix} -5 & -8 \\ -10 & -3 \end{bmatrix}$

5 If C_{ij} denotes the cofactor of elements of matrix A and $A = \begin{bmatrix} 1 & -1 & 2 \\ 0 & 2 & -3 \\ 3 & 2 & 4 \end{bmatrix}$ then the value of $C_{31} \times C_{23} =$

(A) 5 (B) 24 (C) -24 (D) -5

6 If $\frac{d}{dx}\left(f(x)\right) = \log x$ then $f(x)$ equals

(A) $-\frac{1}{x} + c$ (B) $x(\log x - 1) + c$ (C) $x(\log x + x) + c$ (D) $\frac{1}{x} + c$

7 Integrate

$$\int_0^{\frac{\pi}{6}} \sec^2\left(x - \frac{\pi}{6}\right)\, dx$$

(A) $\frac{1}{\sqrt{3}}$ (B) $-\frac{1}{\sqrt{3}}$ (C) $\sqrt{3}$ (D) $-\sqrt{3}$

8 The sum of the order and the degree of the differential equation is

$$\frac{d^2y}{dx^2} + \left(\frac{dy}{dx}\right)^3 = \sin y$$

(A) 5 (B) 2 (C) 3 (D) 4

9 The value of p for which the vectors $2\hat{\imath} + p\hat{\jmath} + \hat{k}\ and - 4\hat{\imath} - 6\hat{\jmath} + 26\hat{k}$ are perpendicular to each other is:

(A) 3 (B) -3 (C) $-\frac{17}{3}$ (D) $\frac{17}{3}$

10 If $\vec{a} + \vec{b} = \hat{\imath}\ and\ \vec{a} = 2\hat{\imath} - 2\hat{\jmath} + 2\hat{k}$ then $\left|\vec{b}\right|$ equals

(A) $\sqrt{14}$ (B) 3 (C) $\sqrt{12}$ (D) $\sqrt{17}$

11 Direction cosines of the line $\frac{x-1}{2} = \frac{1-y}{3} = \frac{2z-1}{12}$ are:

(A) $\frac{2}{7}, \frac{3}{7}, \frac{6}{7}$ (B) $\frac{2}{\sqrt{157}}, -\frac{3}{\sqrt{157}}, \frac{12}{\sqrt{157}}$ (C) $\frac{2}{7}, -\frac{3}{7}, -\frac{6}{7}$ (D) $\frac{2}{7}, -\frac{3}{7}, \frac{6}{7}$

12 If $P\left(\frac{A}{B}\right) = 0.3, P(A) = 0.4\ and\ P(B) = 0.8$ then $P\left(\frac{B}{A}\right)$ is equal to

(A) 0.6 (B) 0.3 (C) 0.06 (D) 0.4

13 Find the value of k for which $f(x)$ is continuous function

$$f(x) = \begin{cases} 3x + 5, x \geq 2 \\ kx^2, \quad x < 2 \end{cases}$$

(A) $-\frac{11}{4}$ (B) $\frac{4}{11}$ (C) 11 (D) $\frac{11}{4}$

14 Find the general solution of the differential equation

$$x\,dy - (1 + x^2)dx = dx$$

(A) $y = 2x + \frac{x^2}{3} + c$ (B) $y = 2\log x + \frac{x^3}{3} + c$

(C) $y = \frac{x^2}{2} + c$ (D) $y = 2\log x + \frac{x^2}{2} + c$

15 If $f(x) = a(x - \cos x)$ is strictly decreasing in R, then 'a' belongs to

(A) $\{0\}$ (B) $(0, \infty)$ (C) $(-\infty, 0)$ (D) $(-\infty, \infty)$

16 The corner points of the feasible region in the graphical representation of a linear programming problem are $(2, 72), (15, 20)$ and $(40, 15)$. If $z = 18x + 9y$ be the objective function, then:

(A) z is maximum at (2, 72), minimum at (15, 20)

(B) z is maximum at (15, 20), minimum at (40, 15)

(c) z is maximum at (40, 15), minimum at (15, 20)

(D) z is maximum at (40, 15), minimum at (2, 72)

17 The number of corner points of the feasible region determined by the constraints is:

$$x - y \geq 0, 2y \leq x + 2, x \geq 0; y \geq 0$$

(A) 2 (B) 3 (C) 4 (D) 5

18 The projection of vector $\hat{\imath} - 2\hat{\jmath} + \hat{k}$ on the vector $4\hat{\imath} - 4\hat{\jmath} + 7\hat{k}$ is

(A) $\frac{19}{9\sqrt{6}}$ (B) $\pm\frac{19}{9}$ (C) $\pm\frac{19}{\sqrt{6}}$ (D) $\frac{19}{9}$

Assertion and Reasoning questions: In the following questions, a statement of Assertion (A) is followed by a statement of Reason (R). Choose the correct answer out of the following choices.

(A) Both A and R are true and R is the correct explanation of A.
(B) Both A and R are true and R is not the correct explanation of A.
(C) A is true but R is false.
(D) A is false but R is true.

19 Assertion (A): The function $f: R \to R, f(x) = |x|$ is not injective
Reason (R): The function $f: R \to R, f(x) = |x|$ is not onto.

20 Assertion (A): The function $f(x) = \cos 2x$ has the minimum value -1 in the domain $\left[0, \frac{\pi}{2}\right]$
Reason (R): The function $f(x) = \cos 2x$ is decreasing in $\left[0, \frac{\pi}{2}\right]$

SECTION – B

21 Write the following in the simplest form:

$$\tan^{-1}\left(\frac{\cos x}{1 - \sin x}\right), where -\frac{3\pi}{2} < x < \frac{\pi}{2}$$

OR

Find the value of $\tan^{-1}\left[2\sin\left(2\cos^{-1}\frac{\sqrt{3}}{2}\right)\right]$.

22 Find the intervals in which the function $f(x) = 20 - 9x + 6x^2 - x^3$ is
(a) strictly decreasing (b) strictly increasing.

23 Show that the local maximum value of $x + \frac{1}{x}$ is less than the local minimum value.

24 A man 1.6 m tall walks at the rate of 0.3 m/sec away from the street light which is 4m above the ground. At what rate is the length of the shadow changing?

OR

A spherical balloon is being inflated at the rate of $35\ cm^3/min$. Find the rate of increase of the radius of the balloon when the diameter is 10 cm.

25 Evaluate:

$$\int_{-1}^{1} x^2|x|\, dx$$

SECTION – C

26 Bag I contains 4 white and 2 black balls. Bag II contains 3 white and 5 black balls. One ball is transferred from Bag I to Bag II and then a ball is drawn from bag II. The ball so drawn is found to be black in colour. Find the probability that the transferred ball is black.

27 Find the particular solution of the differential equation given that $y(0) = 0$

$$\frac{dy}{dx} + \sec^2 x \times y = \tan x \times \sec^2 x$$

OR

Solve the differential equation given by

$$x\, dy - y\, dx - \sqrt{x^2 + y^2} dx = 0$$

28 Solve the following linear programming problem graphically:
Maximise $z = 6x + 3y$ subject to the constraints

$$4x + y \geq 80; 3x + 2y \leq 150; x + 5y \geq 115; x \geq 0 \ and \ y \geq 0$$

OR

Solve the following linear programming problem graphically:
Maximise $z = 400x + 300y$ subject to the constraints

$$x + y \leq 200; x \leq 40; x \geq 20; y \geq 0$$

29 Evaluate:

$$\int_{2}^{5} |x - 1| + |x - 3|\ dx$$

OR

$$\int \frac{x^2}{x^4 - x^2 - 12}\ dx$$

30 Find

$$\int \frac{1}{\cos(x - a) \times \cos(x - b)}\ dx$$

31 If $x = a(cos\, t + t\, sin\, t)$ and $y = a(\sin t - t\cos t)$ then find $\frac{d^2y}{dx^2}$.

SECTION – D

32 On the set of integers Z consider the relation $R = \{(a, b): (a - b)\ is\ divisible\ by\ 5\}$. Show that R is an equivalence relation. Write the equivalence class of 4.

OR

The function $f: N \to N$, defined by

$$f(x) = \begin{cases} x + 1, & if\ x\ is\ odd \\ x - 1, & if\ x\ is\ even \end{cases}$$

Show that the function f is both one-one and onto.

33 Using integration find the area bounded by the curve $y^2 = x\ and\ x = 2y + 3$ in the second quadrant and x-axis.

34 Find the foot of the perpendicular and image of the point P(1, 6, 3) with respect to the line $\frac{x}{1} = \frac{y-1}{2} = \frac{z-2}{3}$. Also find the equation of the perpendicular to the line from P.

OR

Find the shortest distance between the lines

$$\frac{x-1}{2} = \frac{y-2}{3} = \frac{z-3}{4}\ and\ \frac{x+1}{5} = \frac{y-2}{2} = \frac{z-2}{0}$$

35 If $A = \begin{bmatrix} 2 & -3 & 5 \\ 3 & 2 & -4 \\ 1 & 1 & -2 \end{bmatrix}$ find A^{-1}. Use A^{-1} to solve the following system of equations

$$2x - 3y + 5z = 11;\ \ 3x + 2y - 4z = -5\ and\ x + y - 2z = -3$$

SECTION – E

Question numbers 36 to 38 carry 4 marks each. This section contains three case study/ Package based questions. All the three questions have three sub parts of marks 1, 1 and 2 respectively.

36 Lakshmi walks 4 km towards west from her home and reaches her friend Reshmi's house. Then together they walk 3 km in a direction 30 East of North and reaches school. Based on the above information (keeping Lakshmi's house as reference) answer the following questions:

36a Find Lakshmi's displacement from her house to Reshmi's house?

36b Find Reshmi's displacement from her house to school?

36c Find Lakshmi's displacement from her house to school?

OR

Locate the position of the school.

37 Three persons A, B, C apply for the manager post in a company, where the chances of their selection is given by the ratio 1:2:4. If A, B, C are selected as manager, the probability that a new product is introduced from the company by them is 0.8, 0.5, 0.3 respectively.

37a What is the probability that a new product is introduced?

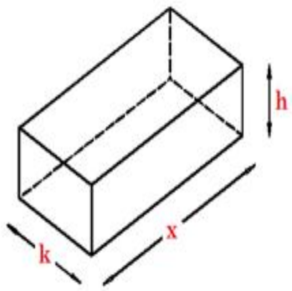
37b What is the probability that a new product is not introduced?

37c If the new product is not introduced, what is the probability that C is selected as manager?

OR

If the new product is introduced, what is the probability that B is selected as manager?

38 A foreign client approaches ISHA Bricks Company for a special type of bricks. The client requests for few samples of bricks as per their requirement. The solid rectangular brick is to be made from 1 cubic feet of clay of special type. The brick must be 3 times as long as it is wide.



38a According to the figure shown, the length of brick is 'x', width is 'k' and height is 'h'. Obtain an expression in terms of 'h' and 'k'.

38b Express the surface area (S) of the brick, as a function of 'k'

38c Find $\frac{dS}{dk}$. At what value of k, $\frac{dS}{dk} = 0$?

Show that $\frac{d^2S}{dk^2}$ is positive, at this obtained value of k. what does it signify?

OR

Find the minimum value of S, using second derivative test.