BK BIRLA CENTRE FOR EDUCATION

SARALA BIRLA GROUP OF SCHOOLS

SENIOR Secondary Co-ed DAY CUM BOYS' RESIDENTIAL SCHOOL

PRE BOARD – 1 EXAMINATION 2023-24

MATHEMATICS (041)

Class: XII Science — Duration: 3 Hrs

Date: 13-12-2023 — **MARKING SCHEME** — Max. Marks: 80

SECTION – A

Question	Answer	Scheme
1	$x = -1$ *and* $y = 3$. *Therefore*, $x + y = 2$	Answer B
2	$\|4\,A^{-1}\| = 16\,\|A^{-1}\| = 16 \times \frac{1}{\|A\|} = 8$	Answer C
3	$\|adj\ A\| = \|A\|^{n-1}$. *Therefore*, $\|A\|^2 = 64$. *so*, $\|A\| = \pm 8$	Answer D
4	$B = \begin{bmatrix} -6 & -8 \\ -10 & -4 \end{bmatrix}$.	Answer B
5	$(-1) \times (-5)$	Answer A
6	Using integration parts $f(x) = \int \log x\ dx = x(\log x - 1) + c$	Answer B
7	$\tan\left(x - \frac{\pi}{6}\right) = 0 - \tan\left(-\frac{\pi}{6}\right) = \frac{1}{\sqrt{3}}$.	Answer A
8	Order = 2 and degree = 1	Answer C
9	$-8 - 6p + 26 = 0$ *or* $-6p + 18 = 0$. Therefore, $p = 3$	Answer A
10	Magnitude of b vector = $\sqrt{1 + 4 + 4} = 3$	Answer B
11	Line L: $\frac{x-1}{2} = \frac{y-1}{-3} = \frac{z-\frac{1}{2}}{6}$. Therefore, direction cosines of $(2, -3, 6)$ are $\left(\frac{2}{7}, -\frac{3}{7}, \frac{6}{7}\right)$	Answer D
12	$P(A \cap B) = P(\frac{A}{B}) \times P(B)$=0.3 x 0.8 = 0.24. So $P\left(\frac{B}{A}\right) = \frac{P(A\cap B)}{P(A)} = \frac{0.24}{0.4} = 0.6$	Answer A
13	$f(x)$ is continuous at $\mathrm{x} = 2$. So, $3 \times 2 + 5 = \mathrm{k} \times 2^2$. $\therefore k = \frac{11}{4}$	Answer D
14	*convert into variable separable form.* $\frac{dy}{dx} = \frac{(2 + x^2)}{x} = \frac{2}{x} + x$	Answer D
15	$f'(x) = a(1 + \sin x)$. f *is decreasing if* $f'(x) < 0$. $a < 0$ *and* $1 + \sin x > 0$	Answer C
16	*At* $(40, 15), z = 720 + 135 = 855$ *and at* $(15, 20), z = 270 + 180 = 450$	Answer C
17	*corner points are* $(0, 0), (0, 1), (2, 2)$.	Answer B
18	Projection = $\frac{4+8+7}{\sqrt{16+16+49}} = \frac{19}{9}$	Answer D
19	Assertion: True Reason: True Reason is not in connection with Assertion	Answer B

20	Assertion is true; Reason is also true. Reason is not supporting assertion	Answer B

SECTION – B

21	$=\tan^{-1}\left(\frac{\left(\cos^2\frac{x}{2}-\sin^2\frac{x}{2}\right)}{\left(\cos\frac{x}{2}-\sin\frac{x}{2}\right)^2}\right)=\tan^{-1}\left(\frac{\cos\frac{x}{2}+\sin\frac{x}{2}}{\cos\frac{x}{2}-\sin\frac{x}{2}}\right)=\tan^{-1}\left(\frac{1+\tan\frac{x}{2}}{1-\tan\frac{x}{2}}\right)=\tan^{-1}\left(\tan\left(\frac{\pi}{4}-\frac{x}{2}\right)\right)=\frac{\pi}{4}-\frac{x}{2}$	1 1
OR	$=\tan^{-1}\left[2\sin\left(2\times\frac{\pi}{6}\right)\right]=\tan^{-1}\left[2\sin\frac{\pi}{3}\right]=\tan^{-1}\left(2\times\frac{1}{2}\right)=\tan^{-1}(1)=\frac{\pi}{4}$	1 1
22	$f'(x)=-9+12x-3x^2=-[3x^2-12x+9]=-3(x-1)(x-3)$. Function is increasing in $(-\infty,1)\cup(3,\infty)$and decreasing $(1,3)$	1 1
23	$f'(x)=1-\left(\frac{1}{x^2}\right)$ and $f'(x)=0\ \rightarrow x=\pm1. f''(x)=\frac{2}{x^3}$. Now minimum value = $(1,2)$ and maximum value = $(-1,-2)$.	1 0.5 0.5
24	$\frac{4}{x+s}=\frac{1.6}{s}$ or $4s=1.6x+1.6s$ or $4s-1.6s=1.6x$ or $2.4s=1.6x$ or $3s=2x$ $3\frac{ds}{dt}=2\frac{dx}{dt}$ or $3\frac{ds}{dt}=2\times0.3=0.6. Therefore, \frac{ds}{dt}=0.2\ m/sec$	1 1
OR	Volume of spherical balloon $V=\frac{4\pi r^3}{3}$; $\frac{dv}{dt}=\frac{4\pi}{3}\times3r^2\frac{dr}{dt}$ or $35=4\pi\times5^2\frac{dr}{dt}$ Therefore, $\frac{dr}{dt}=\frac{7}{20\pi}\ cm/min$	1 1
25	$\int_{-1}^{0}-x^3\,dx+\int_{0}^{1}x^3\,dx=\frac{1}{4}+\frac{1}{4}=\frac{1}{2}$	1 1

SECTION - C

26	$E1=$ white ball is transferred from bag I to bag II and hence $P(E1)=\frac{4}{6}=\frac{2}{3}$ $E2=$ black ball is transferred from bag 1 to bag II and so, $P(E2)=\frac{2}{6}=\frac{1}{3}$ A = black ball is drawn from bag 2. $P\left(\frac{A}{E1}\right)=\frac{5}{9}$ and $P\left(\frac{A}{E2}\right)=\frac{6}{9}$ $P(A)=P(E1)P\left(\frac{A}{E1}\right)+P(E2)P\left(\frac{A}{E2}\right)=\frac{2}{3}\times\frac{5}{9}+\frac{1}{3}\times\frac{6}{9}=\frac{16}{27}$ $P\left(\frac{E1}{A}\right)=\frac{P(E1)P\left(\frac{A}{E1}\right)}{P(A)}=\frac{\frac{2}{3}\times\frac{5}{9}}{\frac{16}{27}}=\frac{10}{16}=\frac{5}{8}$	1 1 1
27	$P=\sec^2 x$ and $Q=\tan x\sec^2 x$. Now integrating factor = $e^{\int p\,dx}=e^{\tan x}$ General solution is $y\,e^{\tan x}=\int\tan x\ \sec^2 x\ e^{\tan x\,dx}$ Therefore, $y=\tan x-1+ce^{-\tan x}$ Particular solution is $y=\tan x-1+e^{-\tan x}$	1 0.5 0.5 1
OR	It is a homogenous differential equation $\frac{dy}{dx}=\frac{y+\sqrt{x^2+y^2}}{x}$ Variable separable form: $\int\frac{dy}{\sqrt{1+v^2}}=\int\frac{dx}{x}$ or $\log\left(v+\sqrt{1+v^2}\right)=\log x+\log c$ Therefore, $\frac{y}{x}+\frac{\sqrt{x^2+y^2}}{x}=cx$	1 1 1
28	Solving $4x+y=80$ and $3x+2y=150$ we get $(2,72)$ as point of intersection. Solving $4x+y=80$ and $x+5y=115$ we get$(15,20)$. Solving $3x+2y=150$ and $x+5y=115$ we get $(85,15)$. Z = 6x + 3y gives 228 at (2, 72); 150 at (15, 20); 555 at (85, 15). So, maximum =555	1 1 1

OR	$Z = 400x + 300y$ is objective function. Solving the in equations we get the corner points as (40, 160) and (20, 180). Now $z = 400x40 + 300x160 = 16000 + 48000 = 64000$ and $z = 400x20 + 300x180 = 8000 + 54000 = 62000$. So, maximum = 64000 at (40, 160)	1 1 1
29	Integration definite: $\int_2^3 2\,dx + \int_3^5 2x - 4\,dx = (2 \times 3 - 2 \times 2) + (25 - 20 - 9 + 12)$ $=6 - 4 + 5 + 3 = 10$ sq. units	1 1 1
OR	Apply partial fraction : $\frac{t}{(t-4)(t+3)} = \frac{\frac{4}{7}}{x^2-4} + \frac{\frac{3}{7}}{x^2+3}$ Integrating, $\frac{4}{7} \times \frac{1}{4} \log\frac{x-2}{x+2} + \frac{3}{7} \times \frac{1}{\sqrt{3}} \tan^{-1}\frac{x}{\sqrt{3}}$	1 1 1
30	Now $\int \frac{1}{\sin(a-b)} \times \frac{\sin(x-b)-\sin(x-a)}{\cos(x-a)\cos(x-b)}\,dx = \frac{1}{\sin(a-b)} \int \tan(x-b) - \tan(x-a)\;dx$ $=\frac{1}{\sin(a-b)} \times [\log\cos(x-a) - \log\cos(x-b)] = \frac{1}{\sin(a-b)} \times \log\left(\frac{\cos(x-a)}{\cos(x-b)}\right) + c$	1 1 1
31	$x = a(\cos t + t\sin t); \frac{dx}{dt} = a(-\sin t + t\cos t + \sin t) = at\cos t$ $y = a(\sin t - t\cos t); \frac{dy}{dt} = a(\cos t + t\sin t - \cos t) = a\,t\sin t$. So, $\frac{dy}{dx} = \tan t$ Now $\frac{d^2y}{dx^2} = \sec^2 t \times \frac{dt}{dx} = \sec^2 t \times \frac{1}{at\cos t} = \frac{(\sec^3 t)}{at}$	1 1 1

SECTION - D

32	For any $a \in Z, a - a = 0$ *is divisible by* 5. Therefore, R is reflexive. For any $a, b \in Z$ *such that* $a - b = 5k$ *for some* $k \in N$ *we have* $b - a = -5k$ It is true throughout. R is symmetric. For any $a, b, c \in Z$ *such that* $a - b = 5m$ *for some* $m \in N; b - c = 5n$ *for some* $n \in N$. Therefore, adding the two results we get $a - c = 5(m + n)$. This implies, R is transitive. Hence R is reflexive, symmetric and transitive. So, R is equivalence relation	1 1 1 1 1
OR	If two numbers are odd: $f(x1) = f(x2) \to x1 + 1 = x2 + 1 \to x1 = x2$ If two numbers are even: $f(x1) = f(x2) \to x1 - 1 = x2 - 1 \to x1 = x2$ Therefore, function is one-one. For every $y \in N$, there exists atleast one preimage $x \in N$ such that $f(x) = y$. It follows the condition that $y + 1$, *if y is odd*; *and* $y - 1$, *if y is even.* Therefore, function is onto.	1 1 1 1 1
33	Solving $y^2 = x$ *and* $x = 2y + 3$ *we get* $(y - 3)(y + 1) = 0$. *when* $y = -1, x = 1$ When $y = 3, x = 9$. The area of the common region = area (parabola) – area (line). $\int_0^9 \sqrt{x} - \frac{x-3}{2}\,dx = \frac{2}{3} \times x^{\frac{3}{2}} - \frac{x^2}{4} + \frac{3x}{2} = \frac{2}{3} \times 27 - \frac{81}{4} + \frac{27}{2} = 18 - 20.25 + 13.5 = 11.75$	1 1 1 1 1
34	Any point on the given line L is: $(\alpha, 2\alpha + 1, 3\alpha + 2)$. Let the common point be M (foot). Now the direction ratios of M are $(\alpha - 1, 2\alpha - 5, 3\alpha - 1)$. Further PM is perpendicular to L. Therefore, dot product is zero. $(\alpha - 1) \times 1 + (2\alpha - 5) \times 2 + (3\alpha - 1) \times 3 = 0$ We get $\alpha = 1$. So, foot of the perpendicular M = (1, 3, 5). Image of the point $P(1, 6, 3)$ *is* $(1, 0, 7)$ using midpoint formula. Equation of the line PM is $\frac{x-1}{0} = \frac{y-6}{6} = \frac{z-3}{-4}$	1 1 1 1 1
OR	Distance between the skew lines: $d = \frac{\lvert(a_2-a_1)\cdot(b_1\times b_2)\rvert}{\lvert b_1\times b_2\rvert}$ Numerator part = $\begin{vmatrix} -2 & 0 & -1 \\ 2 & 3 & 4 \\ 5 & 2 & 0 \end{vmatrix} = -2(0-8) + 0 - 1(4-15) = 16 + 11 = 27$ Denominator part = $= i(0-8) - j(0-20) + k(4-15) = -8i + 20j - 11k$ Therefore, $d = \frac{27}{\sqrt{64+400+121}} = \frac{27}{\sqrt{585}} = \frac{9}{\sqrt{65}}$.	1 1 1 1 1

35	Matrix equation is $AX = B$ and hence $X = A^{-1} \times B$ Now adjoint of A = $\begin{bmatrix} 0 & -1 & 2 \\ 2 & -9 & 23 \\ 1 & -5 & 13 \end{bmatrix}$ and determinant of A = -1 Therefore, $X = -1 \begin{bmatrix} 0 & -1 & 2 \\ 2 & -9 & 23 \\ 1 & -5 & 13 \end{bmatrix} \times \begin{bmatrix} 11 \\ -5 \\ -3 \end{bmatrix} = -1 \begin{bmatrix} -1 \\ -2 \\ -3 \end{bmatrix}. so, x = 1, y = 2, z = 3$	1 1 1 1 1

Case study questions:

36a	Lakshmi's displacement = $-4i$	1
36b	Reshmi's displacement = 3 × $\left[\frac{1}{2}i + \frac{\sqrt{3}}{2}j\right] = \frac{3}{2}i + \frac{3\sqrt{3}}{2}j$	1
36c	Lakshmi's displacement from her house to school = $-4i + \frac{3}{2}i + \frac{3\sqrt{3}}{2}j = -\frac{5}{2}i + \frac{3\sqrt{3}}{2}j$	1 1
OR	Locate the position of the school about origin = $\sqrt{\frac{25}{4} + \frac{27}{4}} = \sqrt{13}\, units$	2
37a	Probability that the new product is introduced = $P(A) = P(E1)P\left(\frac{A}{E1}\right) + P(E2)P\left(\frac{A}{E2}\right) + P(E3)P\left(\frac{A}{E3}\right) = \frac{1}{7} \times 0.8 + \frac{2}{7} \times 0.5 + \frac{4}{7} \times 0.3 = \frac{3}{7}$	1
37b	Probability that the new product is not introduced = 1 – P(new product is introduced) = 1 – $\frac{3}{7} = \frac{4}{7}$	0.5 0.5
37c	Probability of new product is not introduced, from the manager C = $P\left(\frac{C}{A'}\right) = \frac{\frac{4}{7} \times 0.7}{\frac{4}{7}} = 0.7$	2
OR	Probability of new product is introduced, from the manager B = $P\left(\frac{B}{A}\right) = \frac{\frac{2}{7} \times 0.5}{\frac{3}{7}} = \frac{1}{3}$	2
38a	Volume of the brick = $V = xkh = (3k)kh = 3k^2h$. Volume = 1 cubic feet $1 = 3k^2h$	1
38b	$surface\ area\ of\ the\ brick = S = 2(kx + xh + hk) = 2\left(3k^2 + \frac{4}{3k}\right)$	1
38c	Critical value =$\frac{ds}{dk} = 0 \rightarrow 2\left(6k - \frac{4}{3k^2}\right) = 0$. Therefore, $k^3 = \frac{2}{9}$ Now $\frac{d^2s}{dt^2} = 2\left(6 + \frac{8}{3k^3}\right)$. Therefore, at $k = \frac{2}{9}, \frac{d^2s}{dt^2} > 0$. It signifies, S is minimum at this value.	1 1
OR	Minimum value of S = $6\left(\frac{2}{9}\right)^{\frac{2}{3}} + \frac{8}{4 \times \left(\frac{2}{9}\right)^{\frac{1}{3}}} = \frac{48}{9} + \frac{2}{0.695} = 12.9 \approx 13\ square\ units$	1 1