# BK BIRLA CENTRE FOR EDUCATION

**SARALA BIRLA GROUP OF SCHOOLS**

**SENIOR Secondary Co-Ed DAY CUM BOYS' RESIDENTIAL SCHOOL**

## Pre board 1 EXAMINATION 2023-24

**APPLIED MATHEMATICS (241)**

Class: XII Commerce — Duration: 3 Hrs.

Date: 21-12-2023 — MARKING SCHEME — Max. Marks: 80

| Question | Answer | Scheme |
|---|---|---|
| 1 | When $a \equiv b \ (mod\ n)$ implies $a - b = kn\ for\ some\ n\ \in N$.<br>Here, $x - 17 = k \times 4\ and\ x > 18\ and\ \leq 25$.<br>Therefore, the least value that satisfies this statement is $x = 21$ | Answer B |
| 2 | Given $31 \equiv b\ (mod\ 5)$. this means $31 - b = 5 \times k$. Here,<br>$31 - 21 = 5 \times 2, 31 - 31 = 5 \times 0\ and\ 31 - 16 = 5 \times 3.$<br>Therefore, all these statements are satisfied by b. | Answer D |
| 3 | Total number of students = 1000.Let the number of girls be $x$. Therefore,<br>$70\%of\,(1000 - x) + 80\%of\ x = 76\%\ of\ 1000$<br>Hence $70000 - 70x + 80x = 76000\ or\ 10x = 6000\ or\ x = 600.$ | Answer B |
| 4 | Amount that pipe A can fill in 1 minute $=\frac{1}{12}$. Amount that pipe B can fill in 1 minute $=\frac{1}{18}$.<br>When both pipes are open, Amount that filled in 1 minute $= \frac{1}{12} + \frac{1}{18} = \frac{5}{36}$<br>Therefore, time taken to fill the tank is $\frac{36}{5} =$ 7.2 minutes | Answer C |
| 5 | I = $\int \frac{x+3+2-2}{x+5}\,dx = \int 1 - \frac{2}{x+5}\,dx = x - 2\log(x+5) + c$ | Answer B |
| 6 | Given $y = x$. Now $\frac{dy}{dx} =$ 1 and further $\frac{d^2y}{dx^2} = 0$ | Answer B |
| 7 | Property: sum of all probabilities of elementary events = 1<br>Therefore, $0.1 + k + 2k + 2k + k = 1\ or\ 6k = 0.9\ and\ so\ k = \frac{0.9}{6} = \frac{9}{60} = 0.15$ | Answer C |
| 8 | A die is thrown 6 times. Let X be a random variable of getting an even number. Probability of success $= \frac{1}{2}$. Therefore, $P(x = 2) = 6C_2 \times \left(\frac{1}{2}\right)^2 \times \left(\frac{1}{2}\right)^4 = \frac{15}{64}$ | Answer B |
| 9 | Mean of a random variable $= np$ and Variance of the random variable is $npq$. Given n = 4 and $p = \frac{1}{3}$. Therefore, mean $= \frac{4}{3}$ and Variance $= \frac{8}{3}$ | Answer D |
| 10 | A set of observations recorded at an equal interval of time is called **Time Series data** | Answer D |
| 11 | The number of components does a time series data have = 3 | Answer C |
| 12 | Mean $(\mu_a)$before campaign = Mean $(\mu_b)$after campaign | Answer A |
| 13 | Sum of squares of residuals or errors must be MINIMUM | Answer C |
| 14 | Present value of a sequence of payments made at the end of each 6 months @ Rs.60 is<br>= $P = \frac{R}{i} = \frac{60}{0.02} = Rs. 3000$ | Answer A |
| 15 | The common shaded region taking all constraints on a graph sheet is known as Feasible region | Answer D |

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| 16 | Annual depreciation = (cost of the machine – scrap value) / number of life years. Therefore, Annual depreciation = $\frac{40000-8000}{4}=\frac{32000}{4}=Rs.8000$ | Answer A |
| 17 | The total number of samples = 50 bulbs. 15 bulbs are Bajaj, 17 bulbs are Surya and 18 bulbs are Crompton. The point estimate of population proportion of Surya = $\frac{17}{50}=\frac{34}{100}=0.34$ | Answer B |
| 18 | Investment on one share of worth Rs.100 is Rs.80. Rate of dividend = 12%<br>Therefore, effective rate of return ERR = $\frac{return}{investment}\times 100=\frac{12}{80}\times 100=15\%$ | Answer C |
| 19 | Assertion: TRUE. Demand raises on the respective seasons.<br>Reason: TRUE. Seasonal changes are considered within a period of an year | Answer A |
| 20 | $R(x)=3x^2+36x+5$. Therefore, marginal revenue =$\frac{dR}{dx}=6x+36$.<br>The marginal revenue when $x\ =\ 5\ is\ 6(5)+36\ =\ 66$. So, Assertion is TRUE.<br>Reason: TRUE and reason supports the assertion statement | Answer A |

SECTION – B VERY SHORT ANSWER 2 marks each

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| 21 | Data given are 5, 8, 10, 7, 10, 14. The point estimation of the population mean =Sum of all observations/number of observations = $\frac{54}{6}=\ 9$<br>The point estimate of the population standard deviation =<br>$\sqrt{\frac{\sum(x-M)^2}{n-1}}=\sqrt{\frac{16+1+1+4+1+25}{5}}=\sqrt{9.6}=3.09$ | 1<br>0.5<br>0.5 |
| 22 | Let the number of model A products = x and the number of model B products = y. Now the objective function $z\ =\ 8000x\ +\ 12000\ y$ subject to the constraints $9x\ +\ 12y\ \le 180$ and $x+3y\ \le 30$. The corner points obtained on the graph are (20,0), (12, 6) and (0, 10). | 1<br>1 |
| 23 | Area of a triangle = $\frac{1}{2}\big(x1(y2-y3)+x2(y3-y1)+x3(y1-y2)\big)$<br>=$\frac{1}{2}\{3(2-1)-4(1-8)+5(8-2)\}=\frac{1}{2}\{3+28+30\}=\frac{61}{2}=30.5$ square units | 0.5<br>1<br>0.5 |
| OR | Given $A=\begin{bmatrix}2 & -2\\4 & 3\end{bmatrix}$. Determinant of A = $2\times 3-4\times(-2)=14\neq 0$. Inverse of A exists.<br>Adjugate of A = $\begin{bmatrix}3 & 2\\-4 & 2\end{bmatrix}$. Hence $A^{-1}=\frac{1}{14}\begin{bmatrix}3 & 2\\-4 & 2\end{bmatrix}$ | 1<br>1 |
| 24 | A defeats B by 60 metres in a race = A defeats B by 12 seconds.<br>Therefore, A runs the race of 60 metres in 12 seconds. So, he takes 1 metre in 1/5 seconds.<br>Total distance covered in the race = 500 metres<br>The time A has taken to run 500 metres = 500 x 1/5 = 100 seconds | 0.5<br>0.5<br>0.5<br>0.5 |
| OR | A pump fills a tank in 2 hours. Let a leak empties the tank in x hours.<br>Therefore, quantity of tank filled in 1 hour =<br>$\frac{1}{2}-\frac{1}{x}=\frac{3}{7}\ or\ \frac{x-2}{2x}=\frac{3}{7}\ or\ 7x-14=6x\ or\ x=14$<br>Hence the leakage empties the tank in 14 hours if kept open alone. | 1<br><br>1 |
| 25 | Present value of perpetuity $P=\frac{R}{i}\ or\ i=\frac{R}{P}\ or\ \frac{r}{100}=\frac{R}{P}=\frac{500}{10000}.therefore,\ r=10\%$ | 1<br>1 |

SECTION – C SHORT ANSWER 3 marks each

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| 26 | Cost of new house = Rs.4500000; down payment = Rs.500000; loan = Rs.4000000<br>Therefore, $EMI=\frac{P\times i\times(1+i)^n}{(1+i)^n-1}$<br>$EMI=\frac{4000000\times 0.005\times(1.005)^{300}}{(1.005)^{300}-1}=\frac{20000\times 4.4650}{3.4650}=\frac{89300}{3.4650}=Rs.25772$ | 0.5<br>0.5<br>0.5<br>0.5<br>1 |

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| 27 | Apply partial fraction: $\frac{2x+1}{(x+1)(x-2)} = \frac{A}{x+1} + \frac{B}{x-2}$; we simplify<br>$2x + 1 \equiv A(x-2) + B(x+1)$<br>$\int \frac{\frac{1}{3}}{x-2} + \frac{\frac{5}{3}}{x+1} dx = \frac{1}{3}\log(x-2) + \frac{5}{3}\log(x+1) + c$ | 1<br>1<br>1 |
| OR | Apply partial fraction of integration: $\frac{3x-2}{(x+1)(x-2)^2} = \frac{A}{x+1} + \frac{B}{x-2} + \frac{C}{(x-2)^2}$. Therefore,<br>$3x - 2 \equiv A(x-2)^2 + B(x+1)(x-2) + C(x+1)$. Find A, B and C values<br>$\int \frac{-\frac{5}{9}}{x+1} + \frac{\frac{5}{9}}{x-2} + \frac{\frac{4}{3}}{(x-2)^2} dx = -\frac{5}{9}\log(x+1) + \frac{5}{9}\log(x-2) - \frac{\frac{8}{3}}{(x-2)} + c$ | 0.5<br>0.5<br>0.5<br>0.5<br>1 |
| 28 | Sinking fund amount = Rs.1000000; number of years = 10; r = 5% compounded semi annually. Monthly deposit =<br>$R = \frac{A \times i}{(1+i)^n - 1} = \frac{1000000 \times 0.025}{(1.025)^{20} - 1} = \frac{25000}{1.6386 - 1} = Rs. 39148$ | 1<br>1<br>1 |
| 29 | Definite integral: $\int_1^4 \lvert x-5 \rvert \, dx = \int_1^4 -x + 5 \, dx = -\frac{x^2}{2} + 5x = -8 + 20 + \frac{1}{2} - 5 = 7.5$<br>Because of the turning point is 5, the given interval is below it and hence it has only one definite integral part. | 1<br>1<br>1 |
| OR | The demand function $D(x) = 100 - 8x$. Given when<br>$p_0 = 4, therefore\ 4 = 100 - 8x_0. so, x_0 = 12$. Therefore, consumer surplus =<br>$\int_0^{x_0} D(x)dx - x_0p_0$=<br>$\int_0^{12} 100 - 8x \ dx - 12 \times 4 = 100x - \frac{8x^2}{2} - 48 = 1200 - 4 \times 144 - 48 = 576$ | 1<br>1<br>1 |
| 30 | The equation formed are:<br>$4x + 3y + 2z = 60; 2x + 4y + 6z = 90\ and\ 6x + 2y + 3z = 70$<br>The coefficient matrix = $\begin{bmatrix} 4 & 3 & 2 \\ 2 & 4 & 6 \\ 6 & 2 & 3 \end{bmatrix}$ $and\ \Delta = 4(12-12) - 3(6-36) + 2(4-24) = 10$<br>Cofactor matrix of A = $\begin{bmatrix} 0 & 30 & -20 \\ -5 & 0 & 10 \\ 10 & -20 & 10 \end{bmatrix}$. The solution of the linear equations: $X = A^{-1} \times B$<br>$=\frac{1}{10}\begin{bmatrix} 0 & -5 & 10 \\ 30 & 0 & -20 \\ -20 & 10 & 10 \end{bmatrix} \times \begin{bmatrix} 60 \\ 90 \\ 70 \end{bmatrix} = \frac{1}{10}\begin{bmatrix} 250 \\ 400 \\ 400 \end{bmatrix}$. Solution are $x = Rs. 25; y = Rs. 40; z = Rs. 40$ | 1<br>1<br>1 |
| 31 | Given $y = x^3 \log x$.<br>First derivative: $\frac{dy}{dx} = x^3 \times \frac{1}{x} + \log x \times 3x^2 = x^2 + 3x^2 \log x$<br>Second derivative:<br>$\frac{d^2y}{dx^2} = 2x + 3x^2 \times \frac{1}{x} + \log x \times 6x = 2x + 3x + 6x \log x = 5x + 6x \log x$ | 1<br>1<br>1 |

SECTION – D LONG ANSWER TYPE 5 marks each

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| 32 | Volume of the tank = 4000 cubic cm. $V = x^2y\ or\ y = \frac{4000}{x^2}$<br>Surface area of the tank with square bottom =$S = x^2 + 4xy = x^2 + 4x \times \frac{4000}{x^2}$<br>$=\frac{ds}{dx} = 2x + 16000 \times -\frac{1}{x^2} = 2x - \frac{16000}{x^2}$. If $\frac{ds}{dx} = 0\ then\ x^3 = 8000\ and\ so\ x = 20\ cm$<br>Therefore, dimensions of the tank are $l = 20\ cm, b = 20\ cm\ and\ h = 20\ cm$ | 1<br>1<br>1<br>1<br>1 |

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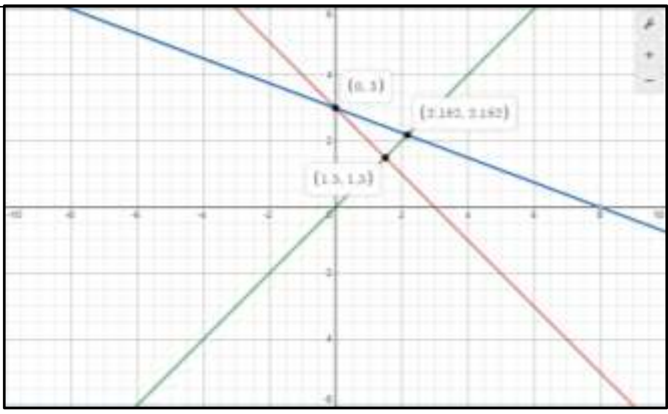
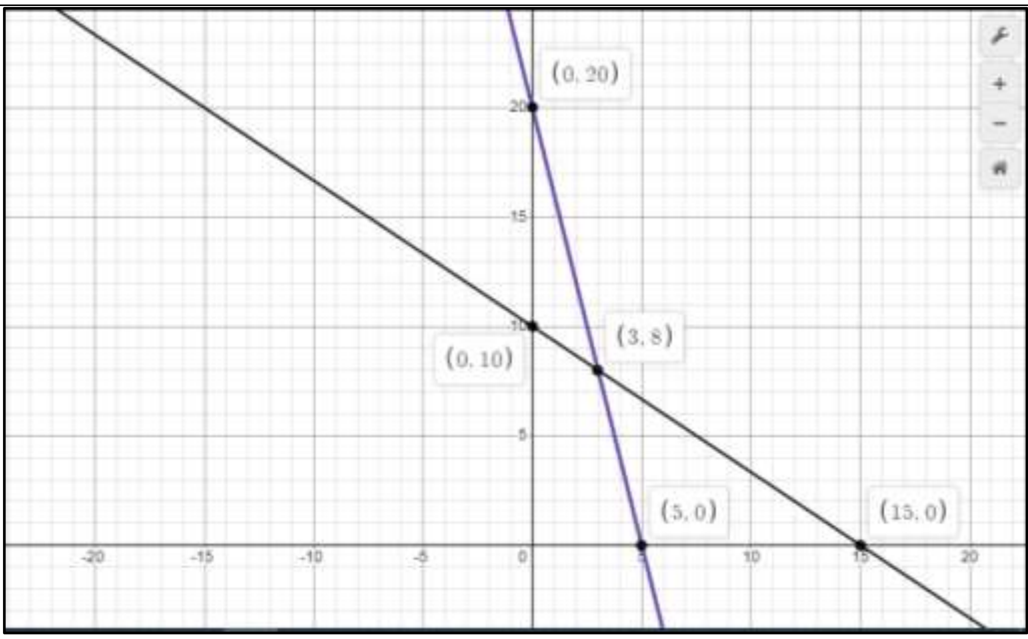
| OR | The equation of the curve is $y^2 = 2x$ and the point is $(1, 4)$. Differentiating with respect to x, we get $2y\frac{dy}{dx} = 2\ or\ \frac{dy}{dx} = \frac{1}{y}$. Therefore, slope of the normal = $-y$. Now equation of the line passing through (1, 4) with slope $-y$ is $y - 4 = -y(x - 1)$. Now solving the curve we get $y - 1 = -y\left(\frac{y^2}{2} - 1\right)\ or\ 2y - 8 = -y^3 + 2y\ or\ y^3 = 8\ and\ hence\ y = 2, x = 2.$<br>Therefore, the nearest point is (2, 2) | 1<br>1<br>1<br><br>1<br>1 |
| 33 | Let X = age among adults in a region; $\mu = 8$; $\sigma = 5$. n=number of adults = 100<br>For $x = 10, z = \frac{10-\mu}{\sigma} = \frac{10-8}{5} = \frac{2}{5} = 0.4$<br>For $x = 14, z = \frac{14-\mu}{\sigma} = \frac{14-8}{5} = \frac{6}{5} = 1.2$ therefore,<br>$\mathrm{P}(10 < \mathrm{X} < 14) = \mathrm{P}(0.4 < \mathrm{Z} < 1.2) = \mathrm{F}(1.2) - \mathrm{F}(0.4) = 0.8849 - 0.6554 = 0.2295$<br>Number of adults in the age group between 10 and 14 years = $0.2295 \times 100 = 22.95 \approx 23$<br>$P(X > 14) = P(Z > 1.2) = 1 - P(Z \leq 1.2) = 1 - 0.8849 = 0.1151$<br>Number of adults in the age more than 14 years = $0.1151 \times 100 = 11.51 \approx 12\ adults$ | 1<br>1<br><br>1<br>1<br>1 |
| OR | Poisson distribution: probability of a bucket produced is defective $p = 0.03$<br>Number of buckets produced $n = 100$. Therefore, $\lambda = np = 100 \times 0.03 = 3$<br>P(zero defective bucket)= $P(X = 0) = \frac{e^{-3} \times 3^0}{0!} = \frac{0.049 \times 1}{1} = 0.049$<br>P(at most one defective bucket) = $P(X \leq 1) = P(0) + P(1) = 0.049 + \frac{e^{-3} \times 3^1}{1!}$<br>Therefore, $P(X \leq 1) = 0.049 + 0.049 \times 3 = 0.049 + 0.147 = 0.196$ | 1<br>1<br>1<br>1<br>1 |
| 34 | At (0, 3), $z = 22 \times 0 + 44 \times 3$<br>$Z = 132$<br>At $(1.5, 1.5), z = 22 \times 1.5 + 44 \times 1.5$<br>$Z = 33 + 66 = 99$<br>At (2.182, 2.182),<br>$Z = 22 \times 2.182 + 44 \times 2.182$<br>=48.004 + 96.008=144.012<br>Maximum of z = 144. The solution region is bounded between (0, 3), (1.5, 1.5) and (2.182, 2.182)<br> | 1<br><br>1<br>1<br><br>1<br><br>1 |
| OR | <br><br>At (15,0),<br>$Z = 18 \times 15 + 10 \times 0 = 270$<br>At (3, 8), $z = 18 \times 3 + 10 \times 8 = 54 + 80 = 134$<br>At (0, 20), $z = 18 \times 0 + 10 \times 20 = 200$<br>Minimum of these z values is 134 and at (3, 8). | 1<br>1<br>1<br>1<br>1 |

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| 35 | The linear equations formed are: $x+y+z = 20;\ 2x+y-z=23;\ 3x+y+z = 46.$<br>Determinant of coefficients = $\begin{vmatrix}1 & 1 & 1\\ 2 & 1 & -1\\ 3 & 1 & 1\end{vmatrix} = (1(1+1)-1(2+3)+1(2-3) = -4$<br>Now $\Delta_1 = \begin{vmatrix}20 & 1 & 1\\ 23 & 1 & -1\\ 46 & 1 & 1\end{vmatrix} = 20(1+1)-1(23+46)+1(23-46) = 40-69-23 = -52$<br>Now $\Delta_2 = \begin{vmatrix}1 & 20 & 1\\ 2 & 23 & -1\\ 3 & 46 & 1\end{vmatrix} = 1(23+46)-20(2+3)+1(92-69) = 69-100+23 = -8$<br>Now $\Delta_3 = \begin{vmatrix}1 & 1 & 20\\ 2 & 1 & 23\\ 3 & 1 & 46\end{vmatrix} = 1(46-23)-1(92-69)+20(2-3) = 23-23-20 = -20$<br>Therefore, $x = \frac{\Delta_1}{\Delta} = -\frac{52}{-4} = 13; y = \frac{\Delta_2}{\Delta} = -\frac{8}{-4} = 2; z = \frac{\Delta_3}{\Delta} = -\frac{20}{-4} = 5$ | 1<br>1<br>1<br>1<br>1 |

SECTION – E CASE STUDY QUESTIONS 4 marks each

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| 36a | The amount filled when pipe A and pipe B are both opened, in 1 hour = $\frac{1}{6}+\frac{1}{8}=\frac{7}{24}$<br>Time taken to fill, when both pipe A and B are opened is $\frac{24}{7} = 3\frac{3}{7}$ hours | 1 |
| 36b | The amount filled when pipe A and pipe C are both opened, in 1 hour =$\frac{1}{6}-\frac{1}{12}=\frac{1}{12}$<br>Time taken to fill, when both pipe A and C are opened is 12 hours | 1 |
| 36c | The amount filled when pipe B and C are both opened, in 1 hour = $\frac{1}{8}-\frac{1}{12}=\frac{1}{24}$<br>Time taken to fill, when both pipe B and C are opened is 24 hours | 1<br>1 |
| OR | The amount filled when all three pipes are opened, in 1 hour = $\frac{1}{6}+\frac{1}{8}-\frac{1}{12}=\frac{5}{24}$<br>Time taken to fill, when all three pipes are opened is $\frac{24}{5} = 4\frac{4}{5}\ hours$ | 1<br>1 |
| 37a | Sum of all probabilities = 1. Therefore, $0.1+k+2k+2k+k=1\ or\ k=\frac{0.9}{6}=0.15$ | 0.5<br>0.5 |
| 37b | P(studies for three hours)= $P\ (x = 3) = k\ (5-3) = 2k = 2 \times 0.15 = 0.3$ | 1 |
| 37c | P(studies for two hours) = $P\ (x = 2) = k \times 2 = 2 \times 0.15 = 0.3$ | 1<br>1 |
| OR | P(studies at least two hours) = $P\ (x \geq 2) = 1-P(x<2) = 1-\{P(0)+P(1)\}$<br>$=1-\{0.1+0.15\} = 0.75$ | 1<br>1 |
| 38a | The forecast for the year 2006 for Urban group is $y = 23+6.9\times 2006$<br>$= 23+13841.4 = 13864.40$ | 0.5<br>0.5 |
| 38b | The forecast for the year 2006 for Rural group is $y = 11.6+5.2\times 2006$<br>$= 11.6+5.2\times 2006 = 11.6+10431.2 = 10442.80$ | 0.5<br>0.5 |
| 38c | The trend line by the method of least squares for Rural Indians group<br>Is $y = 11.6+5.2x$ | 1<br>1 |
| OR | The trend line by the method of least square for Urban Indians group<br>Is $y = 23+6.9x$ | 1<br>1 |