

**PREBOARD 1 EXAMINATION 2023-24**

**MATHEMATICS (041)**

Class : X

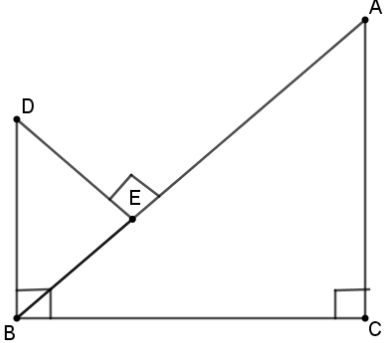
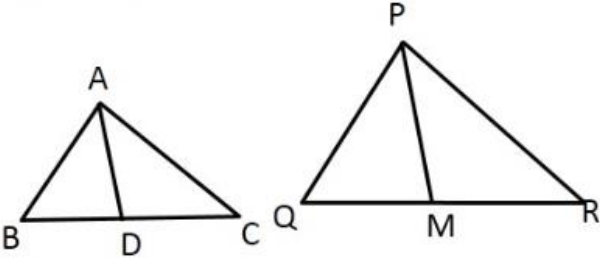
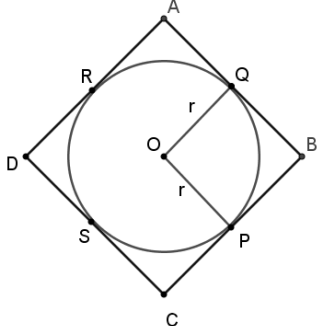
Date : 19/12/23

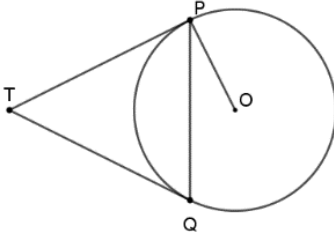
MARKING KEY

Duration : 3 Hrs

Max. Marks : 80

1.	(a) $x^2 - 16$	1
2.	(a) 45 minutes	1
3.	(d) parallel	1
4.	(b) $k \leq 16$	1
5.	(d) $\frac{n}{n+1}$	1
6.	(c) 7	1
7.	(b) $\frac{EF}{RP} = \frac{DE}{PQ}$	1
8.	(d) $x = 3, y = 4$	1
9.	(c) $2\sqrt{3}$ cm	1
10.	(c) 0	1
11.	(a) $\frac{r^2}{4}(\pi - 2)$	1
12.	(b) $360 \text{ cm}^2$	1
13.	(d) 4 : 1	1
14.	(b) multiple of 3	1
15.	(d) (3,5)	1
16.	(b) I and IV	1
17.	(d) 7	1
18.	(a) $y = 3x$	1
19.	(d) (A) is false but (R) is true.	1
20.	(a) Both (A) and (R) are true and (R) is the correct explanation of (A).	1
<b>SECTION B</b>		
<b>Section B consists of 5 questions of 2 marks each.</b>		
21.	$66 = 2 \times 3 \times 11$ $88 = 2^3 \times 11$ $110 = 2 \times 5 \times 11$ HCF = $2 \times 11 = 22$	} 1 } $\frac{1}{2}$

	<p>Total Trees = 264  <math>\therefore</math> Total number of rows = <math>\frac{264}{22} = 12</math></p>	$\frac{1}{2}$	
22.	<p>Sum of zeroes = <math>\alpha + \beta = -(-1) = 1</math>            Product of zeroes = <math>\alpha\beta = -2</math></p> <p>Sum of other zeroes = <math>(2\alpha + 1) + (2\beta + 1) = 2(\alpha + \beta) + 2 = 4</math></p> <p>Product of other zeroes = <math>(2\alpha + 1) \times (2\beta + 1) = 2(\alpha + \beta) + 4\alpha\beta + 1 = -5</math></p> <p><math>\therefore</math> Required polynomial is <math>k(x^2 - 4x - 5)</math></p> <p style="text-align: center;"><b>OR</b></p> <p><math>\alpha + \beta = -\frac{5}{2}, \alpha\beta = \frac{k}{2}</math></p> <p><math>\alpha^2 + \beta^2 + \alpha\beta = \frac{21}{4} \Rightarrow (\alpha + \beta)^2 - \alpha\beta = \frac{21}{4}</math></p> <p><math>\Rightarrow \frac{25}{4} - \frac{k}{2} = \frac{21}{4} \Rightarrow -\frac{k}{2} = -1</math></p> <p><math>\therefore k = 2</math></p>	<p><math>\frac{1}{2}</math></p> <p><math>\frac{1}{2}</math></p> <p><math>\frac{1}{2}</math></p> <p><math>\frac{1}{2}</math></p> <p>1</p> <p><math>\frac{1}{2}</math></p>	
23.	<p><math>\angle DEB = \angle ACB = 90^\circ</math>  <math>\angle ABC = 90^\circ - \angle DBE</math>            Also, <math>\angle BDE = 90^\circ - \angle DBE</math>  <math>\Rightarrow \angle ABC = \angle BDE</math>            So, <math>\triangle BDE \sim \triangle ABC</math></p> <p style="text-align: center;"><b>OR</b></p> <p><math>\triangle ABC \sim \triangle PQR</math>  <math>\Rightarrow \frac{AB}{PQ} = \frac{BC}{QR} = \frac{AC}{PR}</math>  <math>\frac{AB}{PQ} = \frac{2BD}{2QM} = \frac{AC}{PR} \Rightarrow \frac{AB}{PQ} = \frac{BD}{QM}</math></p> <p>Also, <math>\angle B = \angle Q</math> (as <math>\triangle ABC \sim \triangle PQR</math>)            So, <math>\triangle ABD \sim \triangle PQM \Rightarrow \frac{AB}{PQ} = \frac{AD}{PM}</math></p>	  	<p><math>\frac{1}{2}</math></p> <p><math>\frac{1}{2}</math></p> <p><math>\frac{1}{2}</math></p> <p><math>\frac{1}{2}</math></p> <p><math>\frac{1}{2}</math></p> <p><math>\frac{1}{2}</math></p> <p><math>\frac{1}{2}</math></p>
24.	<p><math>DR = DS = 5 \text{ cm}</math>  <math>\Rightarrow AR = AD - DR = 18 \text{ cm}</math>  <math>AQ = AR = 18 \text{ cm}</math>  <math>\Rightarrow QB = 29 - 18 = 11 \text{ cm}</math></p> <p>In quad. <math>OQBP</math>, <math>\angle B = 90^\circ</math> and <math>\angle OQB = \angle OPB = 90^\circ</math>  <math>\therefore OQBP</math> is a square</p> <p>So, <math>r = 11 \text{ cm}</math></p>		<p><math>\frac{1}{2}</math></p> <p><math>\frac{1}{2}</math></p> <p><math>\frac{1}{2}</math></p> <p><math>\frac{1}{2}</math></p>
25.	<p><math>\sin A = \cos A \Rightarrow A = 45^\circ</math>  <math>2\tan^2 A + \frac{1}{\operatorname{cosec}^2 A} + 1 = 2\tan^2 A + \sin^2 A + 1</math></p>	$\frac{1}{2}$	

	$= 2\tan^2 45^\circ + \sin^2 45^\circ + 1$ $= 2(1)^2 + \left(\frac{1}{\sqrt{2}}\right)^2 + 1$ $= 2 + \frac{1}{2} + 1 = \frac{7}{2}$	1 $\frac{1}{2}$
<b>SECTION C</b>		
<b>Section C consists of 6 questions of 3 marks each</b>		
26.	<p>Let us assume that <math>5 + 6\sqrt{7}</math> is rational  Let <math>5 + 6\sqrt{7} = \frac{p}{q}</math>; <math>q \neq 0</math> and <math>p, q</math> are integers  <math>\Rightarrow \sqrt{7} = \frac{p-5q}{6q}</math>  <math>p</math> and <math>q</math> are integers, <math>\therefore p - 5q</math> is an integer  <math>\frac{p-5q}{6q}</math> is a rational number  <math>\Rightarrow \sqrt{7}</math> is a rational number which is a contradiction. So, our assumption that <math>5 + 6\sqrt{7}</math> is a rational number is wrong  Hence <math>5 + 6\sqrt{7}</math> is an irrational number.</p>	$\frac{1}{2}$ 1 $\frac{1}{2}$ $\frac{1}{2}$ $\frac{1}{2}$
27.	<p>Let the length and breadth of rectangle be <math>x</math> m and <math>y</math> m respectively  <math>\therefore (x + 5)(y - 4) = xy - 160</math> and <math>(x - 10)(y + 2) = xy - 100</math>  <math>\Rightarrow 4x - 5y = 140</math> and <math>2x - 10y = -80</math>  Solving, we get <math>x = 60</math> and <math>y = 20</math>  So, length of rectangle = 60 m  Breadth of rectangle = 20 m</p> <p style="text-align: center;"><b>OR</b></p> <p>Let the two numbers be <math>x</math> and <math>y</math> (<math>x &gt; y</math>)  <math>\therefore 2x - 16 = \frac{1}{2}y \Rightarrow 4x - y = 32 \dots (1)</math>  and <math>\frac{1}{2}x - 1 = \frac{1}{2}y \Rightarrow x - y = 2 \dots (1)</math>  Solving, we get <math>x = 10</math> and <math>y = 8</math>  Hence the two numbers are 10 and 8</p>	$\frac{1}{2} + \frac{1}{2}$ $\frac{1}{2} + \frac{1}{2}$ 1  1 1 1 1
28.	<p><math>\angle PTQ = \theta</math>  Now, <math>TP = TQ \Rightarrow TPQ</math> is an isosceles triangle  <math>\angle TPQ = \angle TQP = \frac{1}{2}(180^\circ - \theta) = 90^\circ - \frac{1}{2}\theta</math>  <math>\angle OPT = 90^\circ \Rightarrow \angle OPQ = \angle OPT - \angle TPQ</math>  <math>= 90^\circ - \left(90^\circ - \frac{1}{2}\theta\right) = \frac{1}{2}\theta</math>  <math>= \frac{1}{2}\angle PTQ</math>  So, <math>\angle PTQ = 2\angle OPQ</math></p>	 $\frac{1}{2}$ 1  1  $\frac{1}{2}$
29.	<p>LHS = <math>(\sin^4 \theta - \cos^4 \theta + 1) \operatorname{cosec}^2 \theta</math>  <math>= [(\sin^2 \theta - \cos^2 \theta)(\sin^2 \theta + \cos^2 \theta) + 1] \operatorname{cosec}^2 \theta</math>  <math>= (\sin^2 \theta - \cos^2 \theta + 1) \operatorname{cosec}^2 \theta</math>  <math>= 2 \sin^2 \theta \operatorname{cosec}^2 \theta</math>  <math>= 2</math></p> <p style="text-align: center;"><b>OR</b></p> <p>If <math>\sin x + \operatorname{cosec} x = 2</math>, then find the value of <math>\sin^{19} x + \operatorname{cosec}^{20} x</math>  <math>\sin x + \operatorname{cosec} x = 2</math></p>	1 1  1  1

	$\Rightarrow \sin x + \frac{1}{\sin x} = 2 \Rightarrow \sin^2 x + 1 = 2 \sin x$ $\Rightarrow (\sin x - 1)^2 = 0$ $\therefore \sin x = 1 \Rightarrow \operatorname{cosec} x = 1$ $\text{So, } \sin^{19} x + \operatorname{cosec}^{20} x = 1 + 1 = 2$	<p>1</p> <p>1/2</p> <p>1/2</p>																																												
30.	$\frac{\text{Curved surface area of cylinder}}{\text{curved surface area of cone}} = \frac{8}{5}$ $\Rightarrow \frac{2\pi rh}{\pi rl} = \frac{8}{5}$ $\frac{h}{l} = \frac{4}{5}$ $\frac{h}{\sqrt{h^2+r^2}} = \frac{4}{5}$ $\frac{h^2}{h^2+r^2} = \frac{16}{25}$ $\Rightarrow \frac{r^2}{h^2} = \frac{9}{16}$ $\therefore \frac{r}{h} = \frac{3}{4}$ <p>Hence the ratio of radius and height is 3 : 4</p>	<p>1</p> <p>1/2</p> <p>1/2</p> <p>1/2</p> <p>1/2</p>																																												
31.	<p>(i) <math>P(\text{a face card or a black card}) = \frac{12}{52} + \frac{26}{52} - \frac{6}{52} = \frac{32}{52}</math> or <math>\frac{8}{13}</math></p> <p>(ii) <math>P(\text{neither an ace nor a king}) = 1 - P(\text{either an ace or a king}) = 1 - \left(\frac{4}{52} + \frac{4}{52}\right)</math>  <math>= 1 - \frac{8}{52} = \frac{44}{52}</math> or <math>\frac{11}{13}</math></p> <p>(iii) <math>P(\text{a jack and a black card}) = \frac{2}{52}</math> or <math>\frac{1}{26}</math></p>	<p>1</p> <p>1</p> <p>1</p>																																												
<b>SECTION D</b>																																														
<b>Section D consists of 4 questions of 5 marks each</b>																																														
32.	<table border="1" style="width: 100%; border-collapse: collapse; text-align: center;"> <thead> <tr> <th>Marks</th> <th>Number of students (Cumulative frequency)</th> <th>Frequency</th> <th>Cumulative frequency (less than type)</th> </tr> </thead> <tbody> <tr><td>0 - 10</td><td>80</td><td>3</td><td>3</td></tr> <tr><td>10 - 20</td><td>77</td><td>5</td><td>8</td></tr> <tr><td>20 - 30</td><td>72</td><td>7</td><td>15</td></tr> <tr><td>30 - 40</td><td>65</td><td>10</td><td>25</td></tr> <tr><td>40 - 50</td><td>55</td><td>12</td><td>37</td></tr> <tr><td>50 - 60</td><td>43</td><td>15</td><td>52</td></tr> <tr><td>60 - 70</td><td>28</td><td>12</td><td>64</td></tr> <tr><td>70 - 80</td><td>16</td><td>6</td><td>70</td></tr> <tr><td>80 - 90</td><td>10</td><td>2</td><td>72</td></tr> <tr><td>90 - 100</td><td>8</td><td>8</td><td>80</td></tr> </tbody> </table> <p><math>n = 80 \Rightarrow \frac{n}{2} = 40</math></p> <p><math>\therefore 50 - 60</math> is the median class</p> <p>Median = <math>50 + \frac{40-37}{15} \times 10 = 52</math></p> <p><math>50 - 60</math> is the modal class</p>	Marks	Number of students (Cumulative frequency)	Frequency	Cumulative frequency (less than type)	0 - 10	80	3	3	10 - 20	77	5	8	20 - 30	72	7	15	30 - 40	65	10	25	40 - 50	55	12	37	50 - 60	43	15	52	60 - 70	28	12	64	70 - 80	16	6	70	80 - 90	10	2	72	90 - 100	8	8	80	<p>Correct table – 2</p> <p>1/2</p> <p>1</p> <p>1/2</p>
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50 - 60	43	15	52																																											
60 - 70	28	12	64																																											
70 - 80	16	6	70																																											
80 - 90	10	2	72																																											
90 - 100	8	8	80																																											

$$\text{Mode} = 50 + \frac{15-12}{2 \times 15 - 12 - 12} \times 10 = 55$$

**OR**

Classes	Frequencies ( $f_i$ )	$x_i$	$f_i x_i$
0-30	12	15	180
30-60	21	45	945
60-90	$x$	75	$75x$
90-120	52	105	5460
120-150	$y$	135	$135y$
150-180	11	165	1815
Total	150		$8400 + 75x + 135y$

$$96 + x + y = 150 \Rightarrow x + y = 54 \dots(1)$$

$$\text{Mean} = 91$$

$$\Rightarrow \frac{8400 + 75x + 135y}{150} = 91$$

$$75x + 135y = 5250 \text{ or } 5x + 9y = 350 \dots(2)$$

Solving (1) and (2), we get  $x = 34$  and  $y = 20$

Correct table – 2

$\frac{1}{2}$

1

$\frac{1}{2}$

1

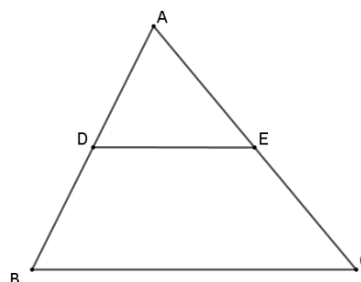
33. For correct statement, given, to prove, figure and construction  
For correct proof

In  $\Delta ABC$ ,  $DE \parallel BC$

$$\Rightarrow \frac{AD}{DB} = \frac{AE}{EC}$$

$$\Rightarrow \frac{x}{x-2} = \frac{x+2}{x-1}$$

Solving, we get  $x = 4$



1

$2\frac{1}{2}$

1

$\frac{1}{2}$

34. Correct Figure

Let speed of car be  $x$  km/h

$$\Rightarrow DC = 12x \text{ m}$$

$$\text{In } \Delta ABC, \frac{AB}{BC} = \tan 30^\circ = \frac{1}{\sqrt{3}}$$

$$\Rightarrow BC = \sqrt{3}AB \dots (1)$$

$$\text{In } \Delta ABD, \frac{AB}{BD} = \tan 45^\circ = 1$$

$$\Rightarrow BD = AB \dots(2)$$

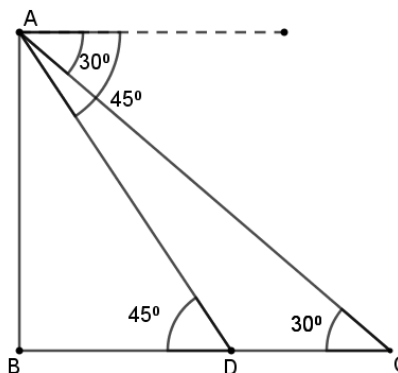
$$\text{Now, } DC = BC - BD \Rightarrow 12x = \sqrt{3}AB - AB = (\sqrt{3} - 1)BD$$

$$BD = \frac{12x}{\sqrt{3}-1}$$

$$\therefore \text{Time taken from D to B } \frac{12x}{\sqrt{3}-1} \times \frac{1}{x} = \frac{12}{\sqrt{3}-1} = 6(\sqrt{3} + 1)$$

$$= 16 \text{ minutes (approx.)}$$

**OR**



1

1

$\frac{1}{2}$

1

$\frac{1}{2}$

$\frac{1}{2}$

$\frac{1}{2}$

	<p style="text-align: center;"><b>Correct Figure</b></p> <p>Let the position of the cloud be E and F be the image of the cloud in the lake  Let <math>ED = h \text{ m}</math>, <math>BD = AC = x \text{ m}</math>  In <math>\triangle BDE</math>, <math>\frac{h}{x} = \tan 30^\circ = \frac{1}{\sqrt{3}}</math>  <math>\Rightarrow x = h\sqrt{3} \dots (1)</math>  In <math>\triangle BDF</math>, <math>\frac{FD}{BD} = \frac{10+(h+10)}{x} = \tan 60^\circ</math>  <math>\Rightarrow \sqrt{3} = \frac{h+20}{\sqrt{3}h}</math> (using (1))  <math>\Rightarrow 3h = h + 20</math>  <math>\therefore h = 10 \text{ m}</math>  So, the height of the cloud from the surface of the lake = <math>(10 + 10) \text{ m}</math>  = <math>20 \text{ m}</math></p>		<p style="text-align: center;">1</p> <p style="text-align: center;">1</p> <p style="text-align: center;"><math>\frac{1}{2}</math></p> <p style="text-align: center;">1</p> <p style="text-align: center;"><math>\frac{1}{2}</math></p> <p style="text-align: center;"><math>\frac{1}{2}</math></p> <p style="text-align: center;"><math>\frac{1}{2}</math></p>
35.	<p>Let the usual speed of the flight be <math>x \text{ km/h}</math>  <math>\therefore \frac{1500}{x} - \frac{1500}{x+250} = \frac{30}{60}</math>  <math>\Rightarrow x^2 + 250x - 750000 = 0</math>  <math>(x - 750)(x + 1000) = 0</math>  <math>\therefore x = 750</math> (Rejecting negative value)  Hence the usual speed of the flight = <math>750 \text{ km/h}</math></p>	<p style="text-align: center;">2</p> <p style="text-align: center;"><math>1\frac{1}{2}</math></p> <p style="text-align: center;">1</p> <p style="text-align: center;"><math>\frac{1}{2}</math></p>	
<b>SECTION E</b>			
<b>Section E consists of 3 Case Studies of 4 marks each</b>			
36.	<p>(i) Let <math>\angle DOA = \theta</math>, then <math>\tan \theta = \frac{AD}{AO} = \frac{\sqrt{3}}{1} \Rightarrow \theta = 60^\circ</math>  <math>\angle DOA = \angle COB = 60^\circ</math>  <math>\angle DOC = 180^\circ - (60^\circ + 60^\circ) = 60^\circ</math></p>	<p style="text-align: center;"><math>\frac{1}{2}</math></p> <p style="text-align: center;"><math>\frac{1}{2}</math></p>	
	<p>(ii) Area of two wooden triangles = <math>2 \times \frac{1}{2} \times 7 \times 7\sqrt{3} = 84.77 \text{ cm}^2</math></p>	<p style="text-align: center;">1</p>	
	<p>(iii) <math>\frac{AO}{DO} = \cos 60^\circ \Rightarrow \frac{7}{DO} = \frac{1}{2}</math>  <math>\Rightarrow DO = 14 \text{ cm}</math>  Area of sector <math>DOC = \frac{60}{360} \times \pi \times 14^2 = 102.67 \text{ cm}^2</math>  <p style="text-align: center;"><b>OR</b></p> <math>\frac{AO}{DO} = \cos 60^\circ \Rightarrow \frac{7}{DO} = \frac{1}{2}</math>  <math>\Rightarrow DO = 14 \text{ cm}</math>  Length of tape required = <math>2 \times 14 + \frac{60}{360} \times 2 \times \pi \times 14 = 42.67 \text{ cm}</math></p>	<p style="text-align: center;">1</p> <p style="text-align: center;">1</p> <p style="text-align: center;">1</p> <p style="text-align: center;">1</p>	
37.	<p>(i) <math>a = 15, d = 5</math>  <math>a_{12} = 15 + 11 \times 5 = 70</math></p>	<p style="text-align: center;"><math>\frac{1}{2}</math></p> <p style="text-align: center;"><math>\frac{1}{2}</math></p>	
	<p>(ii) <math>n = 15</math>  Middle row = 8<sup>th</sup> row  <math>a_8 = 15 + 7 \times 5 = 50</math></p>	<p style="text-align: center;"><math>\frac{1}{2}</math></p> <p style="text-align: center;"><math>\frac{1}{2}</math></p>	
	<p>(iii) <math>1875 = \frac{n}{2} [2 \times 15 + (n - 1) \times 5]</math>  <math>\Rightarrow n^2 + 5n - 750 = 0</math></p>	<p style="text-align: center;"><math>\frac{1}{2}</math></p> <p style="text-align: center;">1</p>	

	$(n + 30)(n - 25) = 0 \Rightarrow n = 25$ $\therefore$ Total number of rows required = 25 <p style="text-align: center;"><b>OR</b></p> $1250 = \frac{n}{2}[2 \times 15 + (n - 1) \times 5]$ $\Rightarrow n^2 + 5n - 500 = 0$ $(n + 25)(n - 20) = 0 \Rightarrow n = 20$ $\therefore$ Number of rows left = $30 - 20 = 10$	$\frac{1}{2}$    $\frac{1}{2}$ $\frac{1}{2}$ $\frac{1}{2}$ $\frac{1}{2}$
38.	(i) $P(3,3), Q(8,2), R(6,5)$ Coordinates of required point are $\left(\frac{3+8+6}{3}, \frac{3+2+5}{3}\right) = \left(\frac{17}{3}, \frac{10}{3}\right)$	1
	(ii) $PR = QR = \sqrt{13}$ $PQ = \sqrt{26}$ $PQ^2 = PR^2 + QR^2$ $\therefore \Delta PQR$ is an isosceles right triangle	$\frac{1}{2}$   $\frac{1}{2}$
	(iii) Area of the plot to row seeds = $13 \times 9 - \frac{1}{2} \times \sqrt{13} \times \sqrt{13}$ $= 110.5 \text{ m}^2$ <p style="text-align: center;"><b>OR</b></p> <div style="text-align: center;"> <math>2 : 3</math>  </div> Coordinates of required point are $\left(\frac{2 \times 8 + 3 \times 13}{2+3}, \frac{2 \times 2 + 3 \times 9}{2+3}\right)$ $= \left(11, \frac{31}{5}\right)$	1 1  1 1