BK BIRLA CENTRE FOR EDUCATION

SARALA BIRLA GROUP OF SCHOOLS

SENIOR SECONDARY | CO-ED DAY CUM BOYS' RESIDENTIAL SCHOOL

MID-TERM EXAMINATION 2023-24

MATHEMATICS (041)

Class: XII Science | Duration: 3 Hrs

Date: 13-10-2023 | Max. Marks: 80

MARKING SCHEME

SECTION – A

Question	Answer	Scheme
1	$f(f(x)) = f\left[(3-x^3)^{\frac{1}{3}}\right] = \left[3-\left((3-x^3)^{\frac{1}{3}}\right)^3\right]^{\frac{1}{3}} = [3-3+x^3]^{\frac{1}{3}} = x$	Answer C
2	R={(2,3), (2,7), (3, 7), (3, 10), (4, 3), (4, 6), (4, 7), (4, 10), (5, 3), (5, 6), (5, 7),} Domain of R = {2, 3, 4, 5}	Answer D
3	$f(x) = (x-1)^2 + 1.\ Since\ (x-1)^2 is\ always\ \geq 0. f(x)\ will\ be\ always \geq 1$	Answer B
4	$cot^{-1}x\ can\ be\ written\ as\ sin^{-1}\left(\frac{1}{\sqrt{1+x^2}}\right)$. Therefore, $\sin\left(sin^{-1}\left(\frac{1}{\sqrt{1-x^2}}\right)\right) = \frac{1}{\sqrt{1-x^2}}$	Answer D
5	$cos^{-1}x = \frac{\pi}{2} - sin^{-1}x = \frac{\pi}{2} - \frac{\pi}{5} = \frac{3\pi}{10}$	Answer A
6	Range of inverse sin function is $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$	Answer B
7	The order of the matrix is 3. $Det(adj\ A) = \det(A)^2$. Therefore, $\|adj\ A\| = 4^2 = 16$	Answer A
8	In general, matrix multiplication is not commutative. Therefore, this statement is true only when $AB = BA$	Answer C
9	Product of (1x3) (3x3) (3x1) matrices leads to 1x1	Answer C
10	Determinant is a number associated to every square matrix	Answer C
11	Given: $A \times A^T = I$. Now $\|A\|\|A^T\| = \|I\|$. But $\det(A) = \det(A^T)$. So, $\det(A^2) = 1$	Answer D
12	$\|3AB\| = 27\|A\|\|B\| = 27 \times (-1) \times 3 = -81$	Answer C
13	$cosec\ x + \cot x = \frac{1+\cos x}{\sin x} = \frac{2cos^2\left(\frac{x}{2}\right)}{2\sin\frac{x}{2}\cos\frac{x}{2}} = \cot\frac{x}{2}$. So, $tan^{-1}\left(\cot\frac{x}{2}\right) = \frac{\pi}{2} - \frac{x}{2} \therefore \frac{dy}{dx} = -\frac{1}{2}$	Answer A
14	$f(x) = x^2 - 4x + 6;\ f'(x) = 2x - 4; f'(x) > 0\ only\ when\ x > 2$	Answer B
15	$\frac{dy}{dx} = \frac{1}{\cos e^x} \times -\sin e^x \times e^x = -e^x \tan e^x$	Answer D
16	$f'(x) = -2\sin x + 1; Now\ f'(x) = 0\ implies\ \sin x = \frac{1}{2}\ and\ \therefore x = \frac{\pi}{6}$. The least value of the function is $f\left(\frac{\pi}{6}\right) = 2 \times \frac{\sqrt{3}}{2} + \frac{\pi}{6}$.	Answer B
17	$\frac{dy}{dx} = -5\sin x - 3\cos x\ \ and\ hence\ \frac{d^2y}{dx^2} = -5\cos x + 3\sin x = -y$	Answer A
18	$Rf'(1) = \lim_{h\to 0}\frac{f(1+h) - f(1)}{h} = \lim_{h\to 0}\frac{[1+h] - [1]}{h} = \frac{1-1}{h} = 0$	Answer A

19	Assertion: True Reason: True Reason is in connection with Assertion	Answer A
20	Assertion is true; but reason is false.	Answer C

SECTION – B

21	$=-\frac{\pi}{6}+\frac{\pi}{3}+\left(-\frac{\pi}{4}\right)=\frac{-2\pi+4\pi-3\pi}{12}=\frac{-\pi}{12}$	1 1
OR	$=[x^2-3x+2=(x-1)(x-2). Therefore\ f(x)\geq 0\ only\ when\ D=(-\infty,1]\cup[2,\infty)]$	1 1
22	$y^2=\sin x+y; 2y\frac{dy}{dx}=\cos x+\frac{dy}{dx};\ \therefore\ \frac{dy}{dx}=\frac{\cos x}{2y-1}$	1 1
23	One number is x and the other is $24-x$. Now $P=x(24-x); \frac{dp}{dx}=x(-1)+(24-x)1=24-2x.\ when \frac{dp}{dx}=0\ we\ get\ x=12. \therefore P\ is\ maximum\ when\ x=12; Max\ P=144$	1 0.5 0.5
24	$3-x^2=3-8; 3-x^2=-5; 3+5=x^2. That\ is, x^2=8;\ \therefore x=2\sqrt{2}$	1 1
25	$\begin{bmatrix}a & b\\ c & d\end{bmatrix}\begin{bmatrix}1 & 2 & 3\\ 4 & 5 & 6\end{bmatrix}=\begin{bmatrix}-7 & -8 & -9\\ 2 & 4 & 6\end{bmatrix}; or \begin{bmatrix}a+4b & 2a+5b & 3a+6b\\ c+4d & 2c+5d & 3c+6d\end{bmatrix}=\begin{bmatrix}-7 & -8 & -9\\ 2 & 4 & 6\end{bmatrix}$ $2a+8b=-14\ and\ 2a+5b=-8\ gives\ b=-2\ and\ a=1$ $2c+8d=4\ and\ 2c+5d=4\ gives\ d=0\ and\ c=2$	1 1
OR	Det (A) $=\|A\|=-24$. Now $\|adj\ A\|=\|A\|^2=(-24)^2=576$	1 1

SECTION - C

26	$\frac{dx}{dt}=-2\sin t+2\sin 2t; \frac{dy}{dt}=2\cos t-2\cos 2t; Now\ \frac{dy}{dx}=\frac{\frac{dy}{dt}}{\frac{dx}{dt}}=\frac{2\cos t-2\cos 2t}{-2\sin t+2\sin 2t}$ $=\frac{\cos t-\cos 2t}{-\sin t+\sin 2t}$	1 1 1
OR	Put $x=\sin t\ and\ consider \cos B=\frac{5}{13}. Therefore \sin B=\frac{12}{13}$ Now $\frac{5x+12\sqrt{1-x^2}}{13}=\frac{5}{13}\sin t+\frac{12}{13}\sqrt{1-\sin^2 t}=\sin t\cos B+\cos t\sin B$ Therefore, $y=sin^{-1}(\sin(t+B))=t+B;\ \therefore y=\sin^{-1}x+B$ So $\frac{dy}{dx}=\frac{1}{\sqrt{1-x^2}}+0=\frac{1}{\sqrt{1-x^2}}$	1 0.5 0.5 1
27	Coefficient matrix = $\begin{bmatrix}1 & -1 & 1\\ 2 & 1 & -3\\ 1 & 1 & 1\end{bmatrix}=A$; Variables=$X=\begin{bmatrix}x\\ y\\ z\end{bmatrix}$; Constants =B= $\begin{bmatrix}4\\ 0\\ 2\end{bmatrix}$ Therefore, matrix equation formed is $AX=B\ and\ hence\ X=A^{-1}B$. Now $\|A\|=10$ $X=\frac{1}{10}\begin{bmatrix}4 & 2 & 2\\ -5 & 0 & 5\\ 1 & -2 & 3\end{bmatrix}\begin{bmatrix}4\\ 0\\ 2\end{bmatrix}=\frac{1}{10}\begin{bmatrix}16+0+4\\ -20+0+10\\ 4+0+6\end{bmatrix}=\begin{bmatrix}2\\ -1\\ 1\end{bmatrix}$	1 1 1
OR	Determinant = $x(-x^2-1)-\sin\theta(-x\sin\theta-\cos\theta)+\cos\theta(-\sin\theta+x\cos\theta)$ $=-x^3-x+x\sin^2\theta+\sin\theta\cos\theta-\cos\theta\sin\theta+x\cos^2\theta$ $=-x^3-x+x=-x^3$ is independent theta	1 1 1

28	Multiply equation (1) by 3 we get $6x + 9y = \begin{bmatrix} 6 & 9 \\ 12 & 0 \end{bmatrix}$ Multiply equation (2) by 2 we get $6x + 4y = \begin{bmatrix} 4 & -4 \\ -2 & 10 \end{bmatrix}$ Subtracting these equations we get $5y = \begin{bmatrix} 2 & 13 \\ 14 & -10 \end{bmatrix}$ $or\ y = \begin{bmatrix} \frac{2}{5} & \frac{13}{5} \\ \frac{14}{5} & -2 \end{bmatrix}$ Now $2x = \begin{bmatrix} 2 & 3 \\ 4 & 0 \end{bmatrix} - \begin{bmatrix} \frac{6}{5} & \frac{39}{5} \\ \frac{42}{5} & -6 \end{bmatrix} = \begin{bmatrix} \frac{4}{5} & -\frac{24}{5} \\ -\frac{22}{5} & 6 \end{bmatrix}; therefore,\ x = \begin{bmatrix} \frac{2}{5} & -\frac{12}{5} \\ -\frac{11}{5} & 3 \end{bmatrix}$	1 1 1
29	Modulus function: LHL =limit x tends to 2 below, is zero RHL = limit x tends to 2 above, is also zero and f(2) = 0 Therefore, function is continuous at x = 2 LHD = $\lim_{h\to 0} \frac{f(2-h)-f(2)}{-h} = \frac{\lvert 2-h-2\rvert - \lvert 2-2\rvert}{-h} = \frac{\lvert -h\rvert - 0}{-h} = \frac{h}{-h} = -1$ RHD = $\lim_{h\to 0} \frac{f(2+h)-f(2)}{h} = \frac{\lvert 2+h-2\rvert - \lvert 2-2\rvert}{h} = \frac{\lvert h\rvert - 0}{h} = \frac{h}{h} = +1$ We see that $LHD \neq RHD$. Therefore, function is not differentiable at x = 2	1 1 1
30	Given x = 1 is the point of maximum Therefore, $f'(x) = 4x^3 - 124x + a$ Now $f'(1) = 0\ implies\ 4(1)^3 - 124(1) + a = 0; so\ a = 120$	1 1 1
31	Reflexive relation: $xRx\ for\ all\ x \in R\ as\ x - x = 0\ is\ always\ divisible\ by\ 3$ Symmetric relation: $xRy\ for\ any\ two\ x, y\ \in R\ means\ x - y = 3k\ for\ some\ integer\ k$ Similarly, $y - x = 3m\ for\ some\ integer\ m\ so\ that\ xRy\ follows\ yRx\ always$ Transitive relation: $For\ any\ three\ elements\ x, y\ and\ z\ \in R, xRy\ means\ x - y = 3k$ And $y - z = 3m. It\ follows\ x - z = 3(k + m)$ Therefore, R is an equivalence relation	1 1 1

SECTION - D

32	Volume of the right circular cone $V = \frac{1}{3}\pi r^2 h$ becomes V =$\frac{1}{3}\pi(r^2 - x^2)(r + x)$ is accounted as per the figure assumed $\frac{dv}{dx} = \frac{\pi}{3}[(r^2 - x^2)(1) + (r + x)(-2x)] = \frac{\pi}{3}(r^2 - x^2 - 2xr - 2x^2)$ To find the critical point, compare $\frac{dv}{dx}$ with zero, we get $r^2 - 2xr - 3x^2 = 0$ Simplifying further, $(r - 3x)(r + x) = 0$. Therefore, $x = \frac{r}{3}$ is the point of maximum Height of the cone inside the sphere $h = r + x = r + \frac{r}{3} = \frac{4r}{3}$	1 1 1 1 1
OR	Wire of length 28 m is cut into two pieces; they are $x\ and\ 28 - x$ First piece is converted into a square. Side of the square = $\frac{x}{4}$ Second piece is converted into a circle. Radius of the circle = $\frac{28-x}{2\pi}$ Area of both taken together A = $\left(\frac{x}{4}\right)^2 + \pi\left(\frac{28-x}{2\pi}\right)^2 = \frac{x^2}{16} + \frac{(28-x)^2}{4\pi}$ Differentiate A with respect to x we get $\frac{dA}{dx} = \frac{2x}{16} + \frac{2(28-x)(-1)}{4\pi}$ When $\frac{dA}{dx} = 0$ then $\frac{x}{8} = \frac{28-x}{2\pi}$. Further simplified we get $x = \frac{112}{\pi+4}$ Second piece = $28 - x = 28 - \frac{112}{\pi+4} = \frac{28\pi}{\pi+4}$	1 1 1 1 1
33	We rewrite $x = \frac{\cos y}{\cos(a+y)} = \frac{\cos(y+a-a)}{\cos(a+y)} = \frac{\cos(a+y)\cos a + \sin(a+y)\sin a}{\cos(a+y)} = \cos a + \sin a \tan(a + y)$ Differentiating $\frac{dx}{dy} = 0 + \sin a \times \sec^2(a + y) = \frac{\sin a}{\cos^2(a+y)}$ Therefore, $\frac{dy}{dx} = \frac{\cos^2(a+y)}{\sin a}$	1 1 1 1 1

34	Given $A = \begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix}$. Now $A^2 = \begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix} \times \begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix} = \begin{bmatrix} 8 & 5 \\ -5 & 3 \end{bmatrix}$ Now $A^2 - 5A + 7I = \begin{bmatrix} 8 & 5 \\ -5 & 3 \end{bmatrix} - \begin{bmatrix} 15 & 5 \\ -5 & 10 \end{bmatrix} + \begin{bmatrix} 7 & 0 \\ 0 & 7 \end{bmatrix} = \begin{bmatrix} 8-15+7 & 5-5+0 \\ -5+5+0 & 3-10+7 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$	1 1 1 2
OR	Symmetric form: $x = \frac{1}{2}(A + A^T) = \frac{1}{2}\left\{\begin{bmatrix} 6 & -2 & 2 \\ -2 & 3 & -1 \\ 2 & -1 & 3 \end{bmatrix} + \begin{bmatrix} 6 & -2 & 2 \\ -2 & 3 & -1 \\ 2 & -1 & 3 \end{bmatrix}\right\} = \frac{1}{2}\begin{bmatrix} 12 & -4 & 4 \\ -4 & 6 & -2 \\ 4 & -2 & 6 \end{bmatrix}$ Skew symmetric form: $y = \frac{1}{2}(A - A^T) = \frac{1}{2}\begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$ Therefore, $x + y = \begin{bmatrix} 6 & -2 & 2 \\ -2 & 3 & -1 \\ 2 & -1 & 3 \end{bmatrix} = A$ We have proved, every square matrix A can be expressed as sum of symmetric and skew symmetric matrices.	1 1 1 1 1
35	Injective: $f(x) = f(y)$ *implies* $x^2 + x + 1 = y^2 + y + 1$ When simplified we get $x^2 - y^2 = y - x$; *or* $(x - y)(x + y) = y - x$; *or* $x + y = -1$ Here, we are taking elements from N. Therefore, each input has a distinct output. So, f is injective. But for $x = 1, f(x) = 3$ which shows 1 and 2 has no preimage in N. Hence f is not surjective. Consider $f^{-1}(3) = x$ *means* $f(x) = 3$ *or* $x^2 + x + 1 = 3$ We simplify as $x^2 + x - 2 = 0$; *or* $(x + 2)(x - 1) = 0$ x cannot be negative. Therefore, $x = 1$. So, $f^{-1}(3) = 1$	1 1 1 1 1

36a	Equations in terms of x and y are $x - y = 50$; *and* $2x + y = 550$	1
36b	Matrix equation of linear equations is $AX = B$ or $\begin{bmatrix} 1 & -1 \\ 2 & 1 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 50 \\ 550 \end{bmatrix}$	1
36c	Using matrix method, $X = A^{-1}B = \frac{1}{3}\begin{bmatrix} 1 & 1 \\ -2 & 1 \end{bmatrix} \times \begin{bmatrix} 50 \\ 550 \end{bmatrix} = \frac{1}{3}\begin{bmatrix} 50 + 550 \\ -100 + 550 \end{bmatrix} = \begin{bmatrix} 200 \\ 150 \end{bmatrix}$ Area of the plot = $xy = 200\ 150 = 30000\ sq.m.$	1 1
OR	A matrix is said to be singular, if its determinant is zero.	2
37a	Given $xy = 150$ and then area of the printed Area = $P = (x - 3)(y - 2)$	1
37b	Solving the function $P = (x - 3)(\frac{150}{x} - 2)$ we will get maximum usage Now $\frac{dP}{dx} = (x - 3) \times -\frac{150}{x^2} + \left(\frac{150}{x} - 2\right) \times 1 = -\frac{150}{x} + \frac{450}{x^2} + \frac{150}{x} - 2$ Therefore , $\frac{dP}{dx} = 0$ *implies* $\frac{450}{x^2} - 2 = 0$ *or* $x^2 = 225$ *or* $x = 15\ cm; y = 10\ cm$ Area of the printed region = $(x - 3)(y - 2) = 12\ x\ 8 = 96\ sq.cm.$	0.5 0.5
37c	When the value of x = 15 cm the printed area is the maximum	2
OR	Area of the printed region = $(x - 3)(y - 2) = 12\ x\ 8 = 96\ sq.cm.$	2
38a	The sin^{-1} function allows values between -1 and 1 only. That is the domain of the function. The answer of sin 4 must be between them. We have to apply allied angles. Therefore, $\pi - 4$ would be the value that satisfies the domain interval.	1
38b	$sin^{-1}(\sin 12) = sin^{-1}(\sin(12 - 4x)) = 12 - 4x$ $cos^{-1}(\cos 12) = \cos^{-1}(\cos(12 - 4x)) = 4x - 12$ Therefore, sum of the results = 12 -4x + 4x – 12 = 0	1
38c	We know that $cos^{-1}\left(-\frac{1}{2}\right) = \frac{2\pi}{3}$ and further $\sin^{-1}\left(\frac{1}{2}\right) = \frac{\pi}{6}$ Therefore, the result = $\frac{2\pi}{3} + 2 \times \frac{\pi}{6} = \frac{3\pi}{3} = \pi$	1 1
OR	Given $\sec^{-2} 2 = x$. Now $\sec x = 2$. This implies $\cos x = \frac{1}{2}$ *.therefore,* $x = \frac{\pi}{3}$	1 1