BK BIRLA CENTRE FOR EDUCATION

SARALA BIRLA GROUP OF SCHOOLS
SENIOR SECONDARY| CO-ED DAY CUM BOYS' RESIDENTIAL SCHOOL

MID-TERM EXAMINATION 2023-24

MATHEMATICS (041)

Class : X | Duration : 3 Hrs
Date : 09-10-2023 | MARKING KEY | Max. Marks : **80**
Admission No.: | Roll No.:

General Instructions:

1. This Question Paper has 5 Sections A, B, C, D and E.
2. Section A has 20 MCQs carrying 1 mark each
3. Section B has 5 questions carrying 02 marks each.
4. Section C has 6 questions carrying 03 marks each.
5. Section D has 4 questions carrying 05 marks each.
6. Section E has 3 case based integrated units of assessment (04 marks each) with sub- parts of the values of 1, 1 and 2 marks each respectively.
7. All Questions are compulsory. However, an internal choice in 2 Qs of 5 marks, 2 Qs of 3 marks and 2 Questions of 2 marks has been provided. An internal choice has been provided in the 2marks questions of Section E

SECTION A

1	5 (c)	1
2	12.5 (b)	1
3	$x^2 - 3x - 2$ (d)	1
4	parabola (c)	1
5	45^0 (d)	1
6	8 (a)	1
7	5 cm (c)	1
8	50^0 (c)	1
9	10 (c)	1
10	4 : 7 (c)	1
11	8 cm (d).	1
12	Mode = 3 Median – 2 Mean (b)	1
13	Increasing (a)	1
14	12 (b)	1
15	Tan 30^0 (b)	1

16 2 (b) 1

17 $\sqrt{3}$ (a) 1

18 Sin ^{2} A (a) 1

19. **DIRECTION:** In the question number 19 and 20, a statement of assertion (A) is followed by a statement of Reason (R). Choose the correct option

Statement A (Assertion): If product of two numbers is 5780 and their HCF is 17, then their LCM is 340

Statement R(Reason) : HCF is always a factor of LCM

(a) **Both assertion (A) and reason (R) are true and reason (R) is the correct explanation of assertion (A)**

(b) Both assertion (A) and reason (R) are true and reason (R) is not the correct explanation of assertion (A)

(c) Assertion (A) is true but reason (R) is false.

(d) Assertion (A) is false but reason (R) is true. 1

20 Statement A (Assertion): If the co-ordinates of the mid-points of the sides AB and AC of ΔABC are D(3,5) and E(-3,-3) respectively, then BC = 20 units 1

Statement R(Reason) : The line joining the mid points of two sides of a triangle is parallel to the third side and equal to half of it.

(a) Both assertion (A) and reason (R) are true and reason (R) is the correct explanation of assertion (A)

(b) Both assertion (A) and reason (R) are true and reason (R) is not the correct explanation of assertion (A)

(c) Assertion (A) is true but reason(R) is false.

(d) Assertion (A) is false but reason(R) is true.

SECTION B

21 Following is the distribution of the long jump competition in which 250 students participated. Find the median distance jumped by the students. Interpret the median

Distance (in m)	0 - 1	1 - 2	2 - 3	3 - 4	4 - 5
Number of Students	40	80	62	38	30

$\frac{n}{2} = \frac{250}{2} = 125$ 1

$\Rightarrow$ median class is 2 - 3, l = 2, h = 1, cf = 120, f = 62

median = $l + \frac{\frac{n}{2} - cf}{f} \times i$

$= 2 + \frac{5}{62}$

$= \frac{129}{62} = 2\frac{5}{62}$ m or 2.08 m 1

50% of students jumped below $2\frac{5}{62}$ m and 50% above it.

OR

The distribution given below shows the runs scored by batsmen in one-day cricket matches. Find the mean number of runs.

Runs scored	0 - 40	40 - 80	80 - 120	120 - 160	160 - 200
Number of batsmen	12	20	35	30	23

Runs scored	Number of batsmen (f_i)	Class mark(x_i)	($f_i x_i$)
0-40	12	20	12 × 20 = 240
40-80	20	60	20 × 60 = 1200
80-120	35	100	35 × 100 = 3500
120-160	30	140	30 × 140 = 4200
160-200	23	180	23 × 180 = 4140
	$\sum f_i = 120$		$\sum f_i x_i = 240 + 1200 + 3500 + 4200 + 4140 = 13280$

2

The mean $\bar{x} = \frac{\sum f_i x_i}{\sum f_i}$

$= \frac{13280}{120}$

$= 110.67$

22 Find the largest number which divides 70 and 125 leaving remainder 5 and 8

Step -1: Subtracting remainders from the given numbers.

As 70 leaves remainder 5 when divided by the required largest number.

⇒The largest number will completely divide 70−5= 65. 1

And 125 leaves remainder 8 when divided by the required largest number.

⇒The largest number will also completely divide 125− 8=117.

Step -2: Finding H.C.F. of 65 and 117.

Factors of 65 =5×13

Factors of 117=3 ×3 × 13

∴The H.C.F. of 65 and 117 is 13.

Hence, 13 is the required largest number. 1

23 Find a relation between x and y such that the point $P(x, y)$ is equidistant from the points A (2, 5) and B (-3, 7).

It is given that: point P(x, y) is equidistant from the points A(2, 5) and B(-3, 7)

⇒ PA = PB

$\Rightarrow \sqrt{(2-x)^2+(5-y)^2} = \sqrt{(-3-x)^2+(7-y)^2}$ $\frac{1}{2}$

$\sqrt{4+x^2-4x+25+y^2-10y} = \sqrt{9+x^2+6x+49+y^2-14y}$

$\sqrt{x^2+y^2-4x-10y+29} = \sqrt{x^2+y^2+6x-14y+58}$ $\frac{1}{2}$

Squaring on both sides

$x^2 + y^2 - 4x - 10y + 29 = x^2 + y^2 + 6x - 14y + 58$ $\frac{1}{2}$

$-10x + 4y - 29 = 0$

Or $10x - 4y + 29 = 0$ ½

24 If in a right angled ΔABC, tan B = $\frac{12}{5}$
, then find sin B.
Given, tan B = $\frac{12}{5}$

so , PERPENDICULAR / BASE = 12/5 hence For figure $\frac{1}{2}$
perpendicular = 12 and base = 5
applying Pythagoras theorem

(hypotenuse) square = (12) square +(5) square $\frac{1}{2}$
(hypotenuse) square = 144 +25
(hypotenuse) square = 169
so (hypotenuse) = 13 $\frac{1}{2}$
Sin B = $\frac{\mathbf{12}}{\mathbf{13}}$ $\frac{1}{2}$

OR

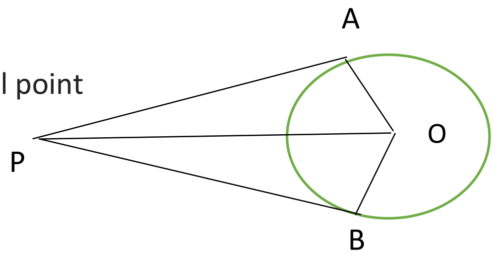
$\frac{1}{Cos\ A} + \frac{Sin\ A}{Cos\ A}$ (1 – Sin A) ½

$\frac{(1+SinA)(1-Sin\ A)}{Cos\ A}$ ½

$\frac{1-Sin^2A}{CosA}$ ½

$\frac{Cos^2A}{CosA}$ = Cos A ½

25 PA = PB --- tangents drawn from an external point
∠APB = 80 ^{0}
∠APO = 40 ^{0} 1
∠POA = 180 – (90 + 40)
= 180 – 130
∠POA = 50 ^{0} 1



SECTION C

26 If the zeroes of the polynomial $x^2 + p x + q$ are double in value to the zeroes of $2x^2 – 5x – 3$, find the value of p and q.

$2x^2 - 5x - 3 = 0$

$2x^2 - 6x + x - 3 = 0$

$(x - 3)(2x + 1) = 0$

$x = 3, \quad -1/2$

Now,

Zeroes of the polynomial $x^2 + px + q$ are double in values to the zeroes of polynomial $2x^2 - 5x - 3$.

Therefore,

Zeroes of polynomial $x^2 + px + q$ will be- 6, −1

Therefore, 1

Sum of roots = −b/a

$6 + (-1) = -p$

$\Rightarrow p = -5$

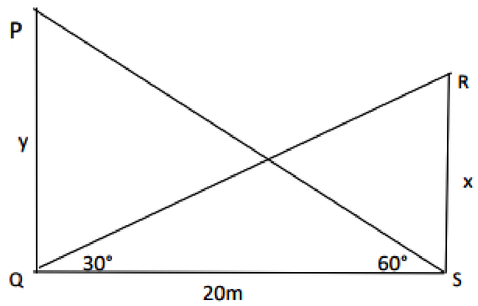
Product of root = c/a

$6 \times -1 = q$

$\Rightarrow q = -6$

Hence the values of p and q are -5 and −6 respectively. 1

27 Two vertical poles of different heights are standing 20m away from each other on the level ground. The angle of elevation of the top of the first pole from the foot of the second pole is 60° and angle of elevation of the top of the second pole from the foot of the first pole is 30°. Find the difference between the heights of two poles. (Take √3 = 1.73) 3



1

$In\ \Delta PQS,\ tan60^0 = \frac{y}{20}$

$\Rightarrow y = 20\sqrt{3}m$ 1

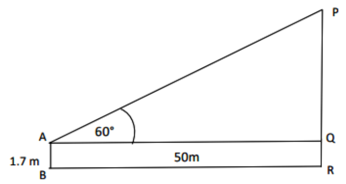
$In\ \Delta RSQ, \quad tan30^0 = \frac{x}{20}$

$\Rightarrow x = \frac{20}{\sqrt{3}}m$ 1

$y - x = 20\sqrt{3} - \frac{20}{\sqrt{3}} = \frac{40}{\sqrt{3}} = \frac{40\sqrt{3}}{3} = 23.06m$

OR

A boy 1.7 m tall is standing on a horizontal ground, 50 m away from a building. The angle of elevation of the top of the building from his eye is 60°. Calculate the height of the building. (Take √3 = 1.73)



1

Let PR be the building and AB be the boy

In ΔPQR, $\tan 60° = \frac{PQ}{50}$

1

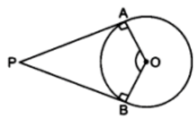
$\sqrt{3} = \frac{PQ}{50}$

$PQ = 50\sqrt{3}$

Height of the building = PR = $(50\sqrt{3} + 1.7)$m = 88.2 m

1

28 Prove that the angle between the two tangents drawn from an external point to a circle is supplementary to the angle subtended by the line segment joining the points of contact to the centre.



Ans:

Given : A circle with centre O

To prove : $\angle APB + \angle AOB = 180^0$ $\frac{1}{2}$

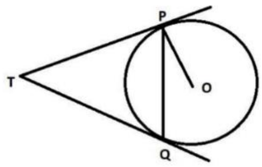
$\angle OAP + \angle OBP + \angle APB + \angle AOB = 360°$ Angle sum property of quadrilateral. 1

$90° + 90° + \angle APB + \angle AOB = 360°$ Tangent perpendicular to radius 1

$\angle APB + \angle AOB = 180°$ $\frac{1}{2}$

OR

Two tangents TP and TQ are drawn to a circle with centre O from an external point T. Prove that $\angle PTQ = 2\angle OPQ$



Ans: Let $\angle PTQ = \theta$

$TP = TQ$ Tangents from an external point. $\frac{1}{2}$

Δ TPQ is an isosceles triangle

$\angle TPQ = \angle TQP$ Base angles $\frac{1}{2}$

$\angle TPQ = \angle TQP \quad = \frac{1}{2}(180^0 - \theta)$

$= 90^0 - \frac{\theta}{2}$ (i) $\frac{1}{2}$

Now , $\angle OPQ = \angle OPT - \angle TPQ$

$= 90^0 - (90^0 - \frac{\theta}{2})$ $\frac{1}{2}$

$\angle OPQ = \frac{\theta}{2}$

$2\angle OPQ = \theta$ $\frac{1}{2}$

$2\angle OPQ = \angle PTQ$ $\frac{1}{2}$

29

Ans : Modal Class : 30-40 , lower limit = 30 , f1= 45 , f0 = 35 and f2 = 25 , h = 10

Mode = $30 + [\frac{45-35}{90-35-25}] \times 10$ 1

$= 30 + [\frac{10}{30}]\, 10$ 1

= 30 + 33.3

Mode = 33.3 1

30 Prove that

$$\frac{\sin\theta - \cos\theta}{\sin\theta + \cos\theta} + \frac{\sin\theta + \cos\theta}{\sin\theta - \cos\theta} = \frac{2}{2\sin^2\theta - 1}$$

Ans: $\frac{(\sin\theta - \cos\theta)^2 + (\sin\theta + \cos\theta)^2}{\text{Sin}^2\theta - \cos^2\theta}$ 1

$= \frac{\sin^2\theta - 2\sin\theta\cos\theta + \cos^2\theta + \sin^2\theta + 2\sin\theta\cos\theta + \cos^2\theta}{\text{Sin}^2\theta - (1 - \sin^2\theta)}$ $1\frac{1}{2}$

$= 2 \sin^2\theta - + 2 \cos^2\theta$

----------------------- $1\frac{1}{2}$

$\text{Sin}^2\theta - 1 + \sin^2\theta$

$= 2 (\sin^2\theta + \cos^2\theta)$

--------------------------- $\frac{1}{2}$

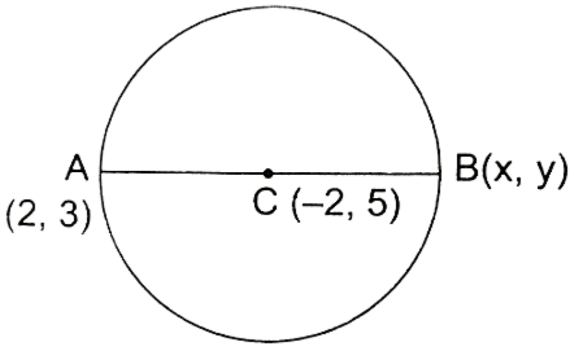
$2 \sin^2\theta - 1$

$= \quad 2$

---------- $\frac{1}{2}$

$2 \sin^2\theta - 1$

31 If the coordinates of one end of a diameter of a circle are (2, 3) and the coordinates of its centre are (-2, 5), then what are the coordinates of the other end of the diameter?



Let C(-2, 5) be the centre of the given circle and A(2, 3) and B(x, y) be the end points of a diameter ACB. 1

Then, C is the midpoint of AB.

$\therefore \frac{2+x}{2} = 2 \text{ and } \frac{3+y}{2} = 5$ 1

$\Rightarrow 2 + x = -4 \text{ and } 3 + y = 10$

$\Rightarrow x = -6 \text{ and } y = 7$

So, the coordinates of B are (-6, 7)` 1

SECTION D

32 An army contingent of 616 members is to march behind an army band of 32 members in a parade. The two groups are to march in the same number of columns. What is the maximum number of columns in which they can march?

Ans :-the definition of HCF states - HCF is the highest number that can be divided exactly into each of two or more numbers. In other words, the HCF of two numbers is the highest number (maximum) that divides both numbers.

Thus, we have the find the HCF of the members in the army band and the army contingent.

HCF (616, 32) will give the maximum number of columns in which they can march. We use Euclid's algorithm to find the H.C.F:

616 = 2 x 2 x 2 x 7 x 11 $1\frac{1}{2}$

32 = 2 x 2 x 2 x 2 x 2 $1\frac{1}{2}$

HCF = 2 X 2 X 2 = 8 1
The HCF (616, 32) is 8. 1

Therefore, the maximum number of columns in which they can march is 8.

OR

Two tanks contain 504 and 735 litres of milk respectively . Find the maximum capacity of a container which can measure the milk of both tank an exact number of times.
Ans: Resolving 504 and 735 into prime factors

504 = 2 x 2 x 2 3 x 3 $1\frac{1}{2}$

735 = 3 x 5 x 7 $1\frac{1}{2}$

Common factors = 3 x 7 1
= 21
HCF = 21
Capacity of the required container = 21 litres. 1

33 The lengths of 40 leaves in a plant are measured correctly to the nearest millimetre, and the data obtained is represented as in the following table: Find the median length of the leaves. 5

Length (mm)	Number of leaves
118-126	3
127-135	5
136-144	9
145-153	12
154-162	5
163-171	4
172-180	2

n = 40 2

⇒ n/2 = 20

But 20 comes under the cumulative frequency 29 and the class-interval against the cumulative frequency 29 is 144-5 – 153-5. So, it is the median class.

∴ l = 144.5, cf = 17, f = 12, and h = 9

Median = l + $\left(\frac{\frac{n}{2}-cf}{f}\right)$ × h 2

= 144.5 + $\left(\frac{20-17}{12}\right)$ × 9

= 144.5 + $\frac{3\times 9}{12}$ = 144.5 + 9/4

= 144.5 + 2.25 = 146.75

1

Hence, median length of leaves = 146-75 mm.

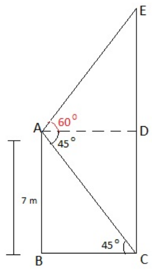
34. From the top of a 7 m high building, the angle of elevation of the top of a cable tower is 60°, and the angle of depression of its foot is 45°. Determine the height of the tower. **5**

From a point on the ground, the angles of elevation of the bottom and the top of transmission tower fixed at the top of a 20 m high building are 45° and 60°, respectively. Find the height of the tower.

Ans: 34

AB be the building's height of 7 m and EC be the height of the tower.
A is the point from where the elevation of the tower is 60°, and the angle of depression of its foot is 45°.

EC = DE + CD
Also, CD = AB = 7 m. and BC = AD figure – 1 1/2



In the right ΔABC, tan 45° = AB/BC
1= 7/BC
BC = 7

BC = AD = 7 1

tan 60° = DE/AD 1

√3 = DE/7 $\frac{1}{2}$

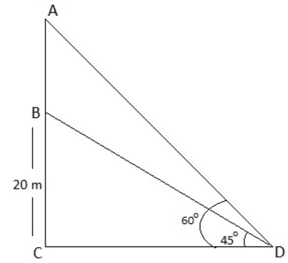
⇒ DE = 7√3 m

Now: EC = DE + CD

Height of tower = (7√3 + 7) = 7(√3+1) 1

OR

From a point on the ground, the angles of elevation of the bottom and the top of transmission tower fixed at the top of a 20 m high building are 45° and 60°, respectively. Find the height of the tower.



Ans: In the right ΔBCD, figure $1\frac{1}{2}$

tan 45° = BC/CD

1 = 20/CD

CD = 20 1

In the right ΔACD,

tan 60° = AC/CD

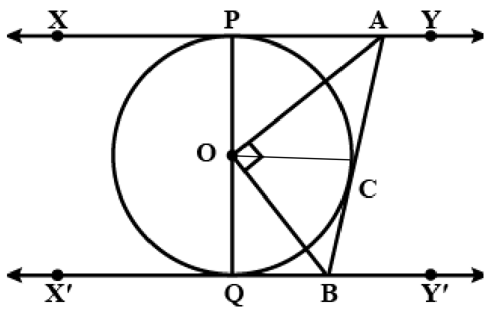
√3 = AC/20

AC = 20√3 $1\frac{1}{2}$

Now, AB = AC – BC = (20√3-20) = 20(√3-1)

Height of transmission tower = 20(√3 – 1) m. 1

35. XY and X’Y’ are two parallel tangents to a circle with centre O and another tangent AB with point of contact C intersecting XY at A and X’Y’ at B. Prove that ∠ AOB = 90°. **5**



Consider the problem

Let us join point O to C

In **ΔOPA and ΔOCA**

OP=OC (Radii of the same circle)
AP=AC (Tangent from point A)

AO=AO (Common side) 2
ΔOPA≅ΔOCA (SSS congruence criterion)

∠POA=∠COA.........(1)

Similarly,

2

ΔQOB ≅ ΔOCB
∠QOB= ∠COB.........(2)

Since , POQ is a diameter of the circle, it is a straight line.

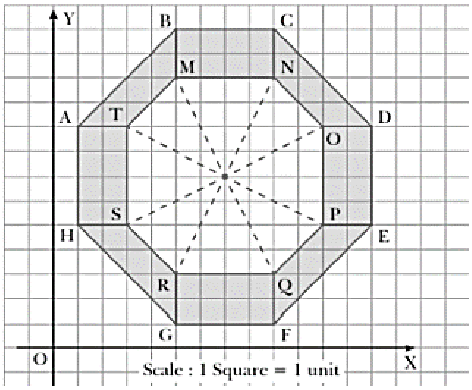
Therefore, ∠POA+∠COA+∠COB+∠QOB=180∘

So, from equation (1) and equation (2)

2∠COA+2∠COB=180 ^{0} ∠COA+∠COB= 90 ^{0}
∠AOB=90 ^{0} 1

SECTION E

36. The top of a table is shown in the figure given below:



(i) Find the coordinates of the points H and G .
Ans: Coordinates of H(1 , 5) , G (5 , 1) 1
(ii) What is the distance between the points A and B is
Ans: Distance between A and B = $\sqrt{(5-1)^2 + (13-9)^2}$ **1**
= $\sqrt{(4)^2 + (4)^2}$ $\frac{1}{2}$
= $\sqrt{32}$ $\frac{1}{2}$
iii) Which among the following have same ordinate?
Ans : Points T and O have same ordinates as 9 **1**

37. Pankaj's father gave him some money to buy avocado from the market at the rate of $p(x) = x^2 - 24x + 128$. Let ∝,β are the zeroes of p(x). Based on the above information, answer the following questions.

i) Find the value of α and β, where α < β

Ans: P(x) = $x^2 - 8x - 16x + 128$ 1

= x (x – 8) – 16 (x- 8) 1/2

X = 8 , 16 Let α = 8 , β 1/**2**

(ii) Find the value of α + β + αβ.

Ans: 8 + 16 + 8 x 16 = 152 **1**

(iii) Find the value of p(2) is

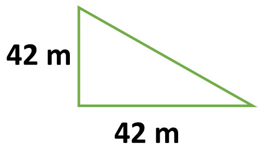
Ans: P(2) = $(2)^2 - 24(2) + 128$

= 4 – 48 + 128 = 84 **1**

38. A group of students of class X visited India Gate on an education trip. The teacher and students had interest in history as well. The teacher narrated that India Gate, official name Delhi Memorial, originally called All-India War Memorial, monumental sandstone arch in New Delhi, dedicated to the troops of British India who died in wars fought between 1914 and 1919. The teacher also said that India Gate, which is located at the eastern end of the Rajpath (formerly called the Kingsway), is about 138 feet (42 metres) in height.

i) What is the angle of elevation if they are standing at a distance of 42m away from the monument?

Ans : **1**



Angle of Elevation = 45 ^{0} **1**

ii) If the altitude of the Sun is at 60^o, find the height of the vertical tower that will cast a shadow of length 20 m .

Ans : 20 √ 3 m **1**

iii) The ratio of the length of a rod and its shadow is 1:1. Find the angle of elevation of the Sun .

Ans: 45 ^{0} **1**

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