# BK BIRLA CENTRE FOR EDUCATION

**SARALA BIRLA GROUP OF SCHOOLS**
**SENIOR SECONDARY CO-ED DAY CUM BOYS' RESIDENTIAL SCHOOL**

## MID-TERM EXAMINATION 2023-24

**MATHEMATICS (041)**

Class: XII Science
Date: 13-10-2023
Admn: _____

Duration: 3 Hrs
Max. Marks: 80
Roll number: __

**General Instructions:**

1 This question paper has 5 sections A, B, C, D and E.
2 Section A has 20 MCQs carrying 1 mark each.
3 Section B has 5 questions carrying 2 marks each.
4 Section C has 6 questions carrying 3 marks each.
5 Section D has 4 questions carrying 5 marks each
6 Section E has 3 case based integrated units of assessment (04 marks each) with subparts of the values 1, 1 and 2 marks each respectively.
7 All questions are compulsory. However, an internal choice in 2 Qs of 5 marks, 2Qs of 3 marks and 2 Qs of 2 marks has been provided. An internal choice has been provided in the 2 marks questions of Section E.
8 Draw neat figures wherever required. Take $\pi = \frac{22}{7}$ wherever required if not stated.

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SECTION – A

1 If $f: R \to R$ be given by $f(x) = (3 - x^3)^{\frac{1}{3}}$ then $fof(x)$ is

(A) $x^{\frac{1}{3}}$ (B) $x^3$ (C) $x$ (D) $(3 - x^3)$

2 A relation R is defined from {2, 3, 4, 5} to {3, 6, 7, 10} by $xRy \Leftrightarrow$ $x$ is relatively prime to $y$. The domain of R is

(A) {2, 3, 5} (B) {3, 5} (C) {2, 3, 4} (D) {2, 3, 4, 5}

3 If $f: R \to A$ given by $f(x) = x^2 - 2x + 2$ is a surjective function, then the set A is

(A) $(1, \infty)$ (B) $[1, \infty)$ (C) $(-\infty, 0)$ (D) $(0, \infty)$

4 If $y = \sin(cot^{-1}x)$ then $y$ is equal to

(A) $(1 + x^2)^{\frac{1}{2}}$ (B) $(1 + x^2)^{-\frac{3}{2}}$ (C) $x$ (D) $(1 + x^2)^{-\frac{1}{2}}$

5 If $sin^{-1}x = \frac{\pi}{5}$ , $x \in [-1, 1]$ then $cos^{-1}x$ is

(A) $\frac{3\pi}{10}$ (B) $\frac{5\pi}{10}$ (C) $-\frac{3\pi}{10}$ (D) $\frac{9\pi}{10}$

6 If $sin^{-1}x = y$ then

(A) $0 \leq y \leq \pi$ (B) $-\frac{\pi}{2} \leq y \leq \frac{\pi}{2}$ (C) $0 < y < \pi$ (D) $-\frac{\pi}{2} < y < \frac{\pi}{2}$

7 Let $A = \begin{bmatrix} a & b & c \\ d & e & f \\ g & h & i \end{bmatrix} = 4$ then |adj A|=

(A) 16 (B) 2 only (C) 4 only (D) 8

8 If A and B are square matrices of order n such that $A^2 - B^2 = (A - B)(A + B)$ then which of the following statement is true?

(A) either A or B is a null matrix (B) $A = B$

(C) $AB = BA$ (D) None of these

9 The order of $\begin{bmatrix} x & y & z \end{bmatrix} \begin{bmatrix} a & h & g \\ h & b & f \\ g & f & c \end{bmatrix} \begin{bmatrix} m \\ n \\ p \end{bmatrix}$ is

(A) $3 \times 1$ (B) $1 \times 3$ (C) $1 \times 1$ (D) $3 \times 3$

10 which of the following is correct:

(A) Determinant is a square matrix

(B) Determinant is a number associated to a matrix

(C) Determinant is a number associated to a square matrix (D) None of these

11 The product of a matrix and its transpose is an identity matrix. The determinant of this matrix is

(A) 0 (B) 1 (C) $-1$ (D) $\pm 1$

12 If A and B are square matrices of order 3 such that $|A| = -1$ and $|B| = 3$, then the determinant of $3AB$ is

(A) 9 (B) 81 (C) $-81$ (D) $-9$

13 The derivative of $tan^{-1}(cosec\, x + \cot x)$ is equal to

(A) $-1/2$ (B) $-1$ (C) 0 (D) 2

14 Find the intervals in which the function $f$ given by $f(x) = x^2 - 4x + 6$ is strictly increasing

(A) $(-\infty, 2) \cup (2, \infty)$ (B) $(2, \infty)$

(C) $(-\infty, 2)$ (D) $(-\infty, 2] \cup [2, \infty)$

15 If $y = log \cos e^x$ then $\frac{dy}{dx}$ is:

(A) $\cos e^{x-1}$ (B) $e^{-x} \cos e^x$ (C) $e^x \sin e^x$ (D) $-e^x \tan e^x$

16 The least value of the function $f(x) = 2\cos x + x$ in the closed interval $\left[0, \frac{\pi}{2}\right]$ is

(A) 2 (B) $\frac{\pi}{6} + \sqrt{3}$ (C) $\frac{\pi}{2}$ (D) does not exist

17 If $y = 5\cos x - 3\sin x$ then $\frac{d^2y}{dx^2}$ is equal to

(A) $-y$ (B) $y$ (C) $25y$ (D) $9y$

18 Let $f(x) = [x]$ where $[x]$ is the greatest integer less than or equal to $x$. Then $Rf'(1) =$

(A) 0 (B) 1 (C) $-1$ (D) not defined

**Assertion and Reasoning questions**: In the following questions, a statement of Assertion (A) is followed by a statement of Reason (R). Choose the correct answer out of the following choices.

(A) Both A and R are true and R is the correct explanation of A.
(B) Both A and R are true and R is not the correct explanation of A.
(C) A is true but R is false.
(D) A is false but R is true.

19 Assertion: $Sec^{-1}(-2) = \frac{2\pi}{3}$.

Reason: $sec^{-1}: A \to B$ where $A = R - (-1,1)$ and $B = \left[0, \frac{\pi}{2}\right) \cup \left(\frac{\pi}{2}, \pi\right]$

20 Assertion: If $A = \begin{bmatrix} 1 & 2 \\ 4 & 9 \end{bmatrix}$ then $A^{-1} = \begin{bmatrix} 9 & -2 \\ -4 & 1 \end{bmatrix}$

Reason: For $A = \begin{bmatrix} a & c \\ d & b \end{bmatrix}$ then $adj\ A = \begin{bmatrix} -a & d \\ c & -b \end{bmatrix}$

## SECTION – B

21 Find the value of $\tan^{-1}\left(-\frac{1}{\sqrt{3}}\right) + \cot^{-1}\left(\frac{1}{\sqrt{3}}\right) + \tan^{-1}\left(\sin(-\frac{\pi}{2})\right)$

**OR**

Find the domain of the function $f: R \to R$ defined by $f(x) = \sqrt{x^2 - 3x + 2}$.

22 If $y = \sqrt{\sin x + y}$ then find $\frac{dy}{dx}$.

23 Find two numbers whose sum is 24 and whose product is as large as possible.

24 Find the value of x for which

$$\begin{vmatrix} 3 & x \\ x & 1 \end{vmatrix} = \begin{vmatrix} 3 & 2 \\ 4 & 1 \end{vmatrix}$$

25 Find the matrix X so that

$$X\begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix} = \begin{bmatrix} -7 & -8 & -9 \\ 2 & 4 & 6 \end{bmatrix}$$

**OR**

Without computing $adj\ A$, find the value of $|adj\ A|$ if $A = \begin{bmatrix} -2 & 0 & 0 \\ 3 & 4 & 0 \\ 10 & -7 & 3 \end{bmatrix}$

## SECTION – C

26 If $x = 2\cos t - \cos 2t \ \ and \ y = 2\sin t - \sin 2t$ then find $\frac{dy}{dx}$.

**OR**

Find $\frac{dy}{dx}$ for the function $y = \sin^{-1}\left(\frac{5x+12\sqrt{1-x^2}}{13}\right)$

27 Solve the following linear equations using matrix method:

$$x - y + z = 4;\ 2x + y - 3z = 0\ and\ x + y + z = 2$$

**OR**

Prove that the determinant is independent of $\theta$:

$$\begin{vmatrix} x & \sin\theta & \cos\theta \\ -\sin\theta & -x & 1 \\ \cos\theta & 1 & x \end{vmatrix}$$

28 Find X and Y if

$$2X + 3Y = \begin{bmatrix} 2 & 3 \\ 4 & 0 \end{bmatrix}\ and\ 3X + 2Y = \begin{bmatrix} 2 & -2 \\ -1 & 5 \end{bmatrix}$$

29 Check the continuity and differentiability of the function $f(x) = |x - 2|$ at $x = 2$.

30 It is given that at $x = 1$, the function $x^4 - 62x^2 + ax + 9$ attains its maximum value, on the interval [0, 2]. Find the value of a.

31 Show that the relation R on the set Z of all integers defined by

$$\{(x, y) \in R \Leftrightarrow (x - y) is\ divisible\ by\ 3\}$$

is an equivalence relation.

## SECTION – D

32 Show that the altitude of the right circular cone of maximum volume that can be inscribed in a sphere of radius r is $\frac{4r}{3}$

**OR**

A wire of length 28 m is to be cut into two pieces. One of the pieces is to be made into a square and the other into a circle. What should be the length of the two pieces so that the combined area of the square and the circle is minimum?

33 If $\cos y = x\cos(a + y)\ where \cos a \neq \pm 1$ then prove that

$$\frac{dy}{dx} = \frac{\cos^2(a+y)}{\sin a}$$

34 If $A = \begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix}$ show that $A^2 - 5A + 7I = 0$

**OR**

Express the following matrix as the sum of a symmetric and a skew symmetric matrix:

$$\begin{bmatrix} 6 & -2 & 2 \\ -2 & 3 & -1 \\ 2 & -1 & 3 \end{bmatrix}$$

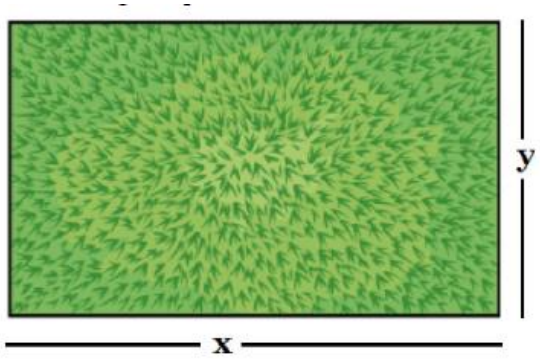
35 Prove that the function $f: N \to N$ is defined by $f(x) = x^2 + x + 1$ is injective but not Surjective. Find the value of $f^{-1}(3)$?

## SECTION – E

36 Bharat wants to donate a rectangular plot of land for a school in his village. When he was asked to give dimensions of the plot, he told that:

(i) If its length is decreased by 50 m and breadth is increased by 50 m, then its area will remain same.

(ii) If its length is decreased by 10 m and breadth is decreased by 20m, then its area will decrease by $5300m^2$.




36a Assume that the length and breadth of the land be x and y (in metres) respectively. Find the equations in terms of x and y.

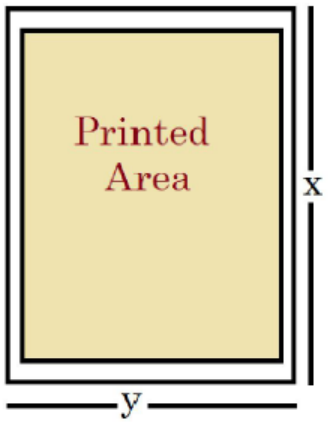
36b Using matrices, represent the linear equations obtained above in 36a.

36c Using matrices, determine the dimensions of the land. Also find the area of the plot of the land.

OR

What is a singular matrix?

37 Following is the pictorial description of a particular page, selected by a school administration.
The total area of the page is 150 $cm^2$
The combined width of the margin at the top and bottom is 3 cm and the side 2 cm.
Using the information given above, answer the following:




37a Find the relation between x and y

37b Find the area of the page where printing can be done.

37c for what value of x, the printable area of the page is maximum? Use derivatives.

OR

What is the area of the printed region?

38 **Inverse trigonometric functions:**

$\text{Sin}(sin^{-1}x) = x \ \ where -1 \leq x \leq 1$ and $sin^{-1}(\sin y) = y \ \ where -\frac{\pi}{2} \leq y \leq \frac{\pi}{2}$

38a Find the value of $sin^{-1}(\sin 4) =?$

38b Find the value of $sin^{-1}(\sin 12) - cos^{-1}(\cos 12) =?$

38c Find the value of

$$cos^{-1}\left(-\frac{1}{2}\right) + 2sin^{-1}\left(\frac{1}{2}\right)$$

$OR$

Find the principal value of $sec^{-1}(2)$