BK BIRLA CENTRE FOR EDUCATION
SARALA BIRLA GROUP OF SCHOOLS
SENIOR SECONDARY CO-ED DAY CUM BOYS' RESIDENTIAL SCHOOL

MID-TERM EXAMINATION 2023-24

APPLIED MATHEMATICS (241)

Class : XII Commerce   Duration: 3 Hrs.
Date : 18-10-2023   Max. Marks: 80
Admn: _____   Roll number: ____

General Instructions:

1 This question paper has 5 sections A, B, C, D and E.
2 Section A has 20 MCQs carrying 1 mark each.
3 Section B has 5 questions carrying 2 marks each.
4 Section C has 6 questions carrying 3 marks each.
5 Section D has 4 questions carrying 5 marks each
6 Section E has 3 case based integrated units of assessment (04 marks each) with subparts of the values 1, 1 and 2 marks each respectively.
7 All questions are compulsory. However, an internal choice in 2 Qs of 5 marks, 2Qs of 3 marks and 2 Qs of 2 marks has been provided. An internal choice has been provided in the 2 marks questions of Section E.
8 Draw neat figures wherever required. Take $\pi = \frac{22}{7}$ wherever required if not stated.

SECTION – A

1 If $A = \begin{bmatrix} 2 & -1 & 3 \\ 4 & 5 & -6 \end{bmatrix}$ and $B = \begin{bmatrix} 1 & 2 \\ -3 & 4 \\ 5 & -6 \end{bmatrix}$ then

   (A) Only AB is defined   (B) only BA is defined
   (C) AB and BA are both defined   (D) AB and BA both are not defined

2 The matrix $\begin{bmatrix} 0 & -5 & 3 \\ 5 & 0 & -7 \\ -3 & 7 & 0 \end{bmatrix}$ is a

   (A) diagonal matrix   (B) symmetric matrix
   (C) skew-symmetric matrix   (D) scalar matrix

3 If A and B are square matrices of same order, then $(A + B)(A - B)$ is equal to
   (A) $A^2 - B^2$ (B) $A^2 - BA - AB + B^2$
   (C) $A^2 - AB + BA - B^2$ (D) $A^2 + AB - BA - B^2$

4 If $A = \begin{bmatrix} 1 & 2 & x \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ and $B = \begin{bmatrix} 1 & -2 & y \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ and $AB = I_3$ then (x + y) equals
   (A) 0   (B) -1 (C) 2   (D) -2

5 If $\begin{vmatrix} 3x & 4 \\ 5 & x \end{vmatrix} = \begin{vmatrix} 4 & -3 \\ 5 & -2 \end{vmatrix}$ then $x =$
   (A) $3\ only$ (B) $-3\ only$ (C) $3\ or\ -3$ (D) $6\ or\ -6$

6   If $\begin{bmatrix} 1 & 3 & 9 \\ 1 & x & x^2 \\ 4 & 6 & 9 \end{bmatrix}$ is a singular matrix, then $x =$

(A)   3   (B)   3 or 6   (C) $3 \; or \frac{3}{2}$ (D) $-3, \frac{3}{2}$

7   If $A = \begin{bmatrix} a & 0 & 0 \\ 0 & a & 0 \\ 0 & 0 & a \end{bmatrix}, a \neq 0$ then $|adj \; A|$ is equal to

(A) a^3 (B) a^9 (C) a^6 (D) a^{27}

8   If $y = \sqrt{x + \sqrt{x + \sqrt{x + \cdots \infty}}}$ then $\frac{dy}{dx} =$

(A) $2y - 1$ (B) $2y + 1$ (C) $\frac{1}{2y-1}$ (D) $\frac{1}{2y+1}$

9   If $x = t^2$ and $y = t^3$ then $\frac{d^2y}{dx^2} =$

(A) $\frac{3}{2}$ (B) $\frac{3}{2}t$ (C) $\frac{3}{2t}$ (D) $\frac{3}{4t}$

10   If $x^2 + xy + y^2 = 0$ then $\frac{dy}{dx}$ is

(A) $\frac{2x+y}{x+2y}$ (B) $-\frac{2x+y}{x+2y}$ (C) $\frac{x+2y}{2x+y}$ (D) $-\frac{x+2y}{2x+y}$

11   Differentiate $log \; x$ with respect to $\frac{1}{x}$ is

(A) $-\frac{1}{x^3}$ (B) $-\frac{1}{x}$ (C) $-x$ (D) $\frac{1}{x}$

12   The equation of the tangent to the curve $y = e^{2x} \; at \; (0, 1)$ is

(A) $y + 1 = 2x$ (B) $1 - y = 2x$ (C) $y - 1 = 2x$ (D)   None of these

13   The smallest value of the polynomial $x^3 - 18x^2 + 96x \; in \; [0, 9]$ is

(A)   128   (B)   135   (C)   160   (D)   0

14   If the selling price of a commodity is fixed at Rs.45 and the cost function $C(x) = 30x + 240$ then the breakeven point is

(A) $x = 10$ (B) $x = 12$ (C) $x = 15$ (D) $x = 16$

15   Which of the following statements are true?

I: The mean of a population is denoted by $\bar{x}$

II: The population mean is a statistic

(A)   I only   (B)   II only   (C)   Both I and II   (D)   None of these

16   The assumed hypothesis which is tested for rejection considering it to be true is called

       (A)   Null hypothesis   (B)   alternative hypothesis
       (C)   simple hypothesis   (D)   true hypothesis

17   A fire in a factory delaying production for some time is

       (A)   Long term trend   (B)   Cyclical trend
       (C)   Seasonal trend   (D)   Irregular trend

18   For the given five values 15, 24, 18, 33, 42, the three years moving averages are

       (A)   19, 22, 33   (B)   19, 25, 31   (C)   19, 30, 31   (D)   19, 25, 33

Assertion and Reasoning questions: In the following questions, a statement of Assertion (A) is followed by a statement of Reason (R). Choose the correct answer out of the following choices.
       (A)   Both A and R are true and R is the correct explanation of A.
       (B)   Both A and R are true and R is not the correct explanation of A.
       (C)   A is true but R is false.
       (D)   A is false but R is true.

19   Assertion: If the demand function of a product is $p = 200 - \frac{x^2}{3}$, then the marginal revenue (MR) of selling 10 units is Rs.120
       Reason: $MR = \frac{d}{dx}(R(x))$

20   Let A be a square matrix of order 2.
       Assertion: $adj(adj\ A) = A$
       Reason: $|adj\ A| = |A|$

SECTION – B

21   If $\begin{bmatrix} x+y & x+2 \\ 2x-y & 16 \end{bmatrix} = \begin{bmatrix} 8 & 5 \\ 1 & 3y+1 \end{bmatrix}$ then write the value of $(y - x)$

22   If A is a square matrix of order 3 and $|A| = 4$, then find the value of $|2A|$.

23   If $y = x^y$, prove that

$$x\frac{dy}{dx} = \frac{y^2}{1 - y\log x}$$

OR

Find the second derivative of $x^3 \log x$

24   A simple random sample of 50 items from a population with $\sigma = 6$ resulted in a sample mean of 32. Provide a 95% confidence interval for the population mean.

25   Obtain the three year moving averages, 3 year centred moving averages for the following series of observations:

Year	1995	1996	1997	1998	1999	2000	2001	2002
Annual Sales (in 10000Rs.)	3.6	4.3	4.3	3.4	4.4	5.4	3.4	2.4

OR

A company produces a commodity with Rs.24000 fixed cost. The variable cost is estimated to be 25% of the total revenue recovered on selling the product at a rate of Rs.8 per unit. Find the cost function and breakeven point.

SECTION – C

26   Find matrices X and Y if
$$2X - Y = \begin{bmatrix} 6 & -6 & 0 \\ -4 & 2 & 1 \end{bmatrix} \text{ and } X + 2Y = \begin{bmatrix} 3 & 2 & 5 \\ -2 & 1 & -7 \end{bmatrix}$$

27   Prove that $\begin{vmatrix} x+y & x & x \\ 5x+4y & 4x & 2x \\ 10x+8y & 8x & 3x \end{vmatrix} = x^3$

OR

Solve the following system of linear equations using Cramer's rule
$$x + y + z + 1 = 0; x + 2y + 3z + 4 = 0 \text{ and } x + 3y + 4z + 6 = 0$$

28   Find the derivative $\frac{dy}{dx}$ for the following implicit function:
$$ax^2 + 2hxy + by^2 + 2gx + 2fy + c = 0$$

29   A cylindrical tank of radius 10 m is being filled with wheat at the rate of 314 cubic metre per hour. Find the rate at which the depth of the wheat is increasing (Use $\pi = 3.14$)

OR

A television manufacturer finds that total cost for the production and marketing x number of television sets is $C(x) = 300x^2 + 4200x + 13500$ and each product is sold for Rs.8400. Determine the breakeven points.

30   Find the equation of the tangent to the curve $ay^2 = x^3$ at the point (am^2, am^3).

31   Consider the following hypothesis test:
$$H_0: \mu = 100 \text{ and } H_a: \mu \neq 100$$
A sample of 65 is used. Compute a range for the p-value and state your conclusion for
$$\bar{x} = 103 \text{ and } S = 11.5. \text{ Use } \alpha = 0.05$$

SECTION – D

32   A survey of 611 office workers investigated telephone answering practices, including how often each office worker was able to answer incoming telephone calls and how often incoming telephone calls went directly to voice mail. A total of 281 office workers indicates that they never need voice mail and are able to take every telephone call.

(i)   What is the point estimate of the population proportion of office workers who are able to take every telephone call?

(ii)   At 90% confidence level, what is the margin of error?

33   Fit a straight line trend by the method of least squares and find the trend value for the year 2008 for the following data:

Year	2001	2002	2003	2004	2005	2006	2007
Production (in lakh tonnes)	30	35	36	32	37	40	36

34   Given that the total cost function for x units of a commodity is:
$$C(x) = \frac{x^3}{3} + 3x^2 - 7x + 16$$
Find the marginal cost (MC); Find the average cost (AC) and Prove that the marginal average cost $MAC = \frac{x(MC) - C(x)}{x^2}$

OR

A square piece of tin of side 18 cm is to be made into a box without top by cutting a square from each corner and folding up the flaps to form an open box. What should be the side of the square to be cut off so that the volume of the box is maximum? Also find this maximum volume.

35   If $y = x^3 \log\left(\frac{1}{x}\right)$ then prove that $x\frac{d^2y}{dx^2} - 2\frac{dy}{dx} + 3x^2 = 0$

OR

Differentiate $x^{\log x} + (\log x)^x$ with respect to x

SECTION – E

36   Product rule of differentiation: $\frac{d}{dx}(uv) = u \times \frac{d}{dx}v + v \times \frac{d}{dx}u$

Quotient rule of differentiation:

$$\frac{d}{dx}\left(\frac{u}{v}\right) = \frac{v \times \frac{d}{dx}u - u \times \frac{d}{dx}v}{v^2}$$

36a   If $y = (x + 2)(x - 3)$ then find $\frac{dy}{dx}$

36b   If $xy^2 = c$ then find $\frac{dy}{dx}$

36c   If $y = \frac{3x-2}{6-4x}$ then find the derivative of $y\left(= \frac{dy}{dx}\right)$.

OR

If $x^2 + y^2 = 1$ then find $\frac{dy}{dx}$

37   A trust invested some money in two type of bonds. The first bond pays 10% interest and second bond pays 12% interest. The trust received Rs.2800 as interest. However, if trust had interchanged money in bonds, they would have got Rs.100 less as interest.

Let the amount invested in first type and second type of bonds be $Rs.\,x$ and $Rs.\,y$ respectively. Based on the above information, answer the following questions:

37a   Write the equations in terms x and y.

37b   If $A = \begin{bmatrix} 5 & 0 \\ 0 & 5 \end{bmatrix}$ then find A^3

37c   Find the amount invested by trust in first and second bonds.

OR

If A is the matrix formed by coefficients of x and y of the given situation, then find the value of $|adj\, A|$

38   A cable network provider in a small town has 500 subscribers and he used to collect Rs.300 per month from each subscriber. He proposes to increase the monthly charges and it is believed from past experience that for every increase of Rs.1, one subscriber will discontinue the service. Based on the above information, answer the following questions:

38a   If $Rs.\,x$ is the monthly increase in subscription amount, then find the number of Subscribers.

38b   Find the total revenue (in Rs.)

38c   Find the number of subscribers which gives the maximum revenue.

OR

Find the maximum revenue generated